# Teacher Knowledge from Mathematics Education Perspective

**Rukiye Gokce**

*Pamukkale Nezihe Derya Baltalı Science and Art Center*

## Introduction

With the development of science and technology, the form of education changes day by day. These changes were especially felt during the pandemic caused by COVID-19.The formal changes of education over time show the importance of the teacher's competence to perform an effective teaching.

Shulman (1987) links his teacher competence to his knowledge and explains this knowledge accumulation according to the "knowledge base / base" theory from the perspective of "knowing and teaching". According to Shulman (1987, s.8), a teacher should have subject matter knowledge, general pedagogical knowledge and pedagogical content knowledge categories.

Shulman (1986) defines the amount and organization of knowledge in a teacher's mind as subject matter knowledge. This category of knowledge requires beyond the knowledge of facts and concepts about the content. General pedagogical knowledge; It is a category of knowledge that covers the principles and strategies of lesson planning process, assessment and classroom management.

Pedagogical content knowledge was first introduced as a knowledge category by Shulman (1986, 1987). Shulman (1986) states that subject matter knowledge will not be sufficient to teach a subject so he refers to "pedagogical content knowledge" (PCK), which is a synthesis of subject matter knowledge and pedagogical knowledge. PCK is "a blend of content and pedagogy that shapes how topics and problems in the teaching offered will be organized, represented and adapted to the interests and abilities of the students."

In Shulman's (1986) explanations for pedagogical content knowledge mentions about ways of shaping and organizing content such as representations, analogies / anaologies, drawings, examples, explanations, demonstration experiments for ideas, understanding the factors that make the content easier or difficult to learn, the readiness of the students, their pre- understanding, and their mistakes and strategies to eliminate the errors in their foresight.

Learning changes depending on the form and quality of presented teaching. A teacher who knows about the readiness, misconceptions and interests of her/his students; How to avoid these mistakes during teaching, predict which method and strategy can be used to provide conceptual understanding in the best practice, and can shape his teaching accordingly. Used representations, explanations, examples, similes are formal factors that make it easier to understand the presented content. These two subcategories pointed out by Shulman (1986) actually determine the line between “knowing very well” and “being able to teach well”. Because, according to Shulman (1987), the capacity of a teacher is about transforming his / her content knowledge into strong forms pedagogically.

International Student Assessment Program administered by the Organization for Economic Cooperation and Development(OECD) every three years to fifteen-year-old students (PISA) and Trends in International Mathematics and Science Study (TIMSS), which is applied by the International Association for the Evaluation of Educational Achievement (IEA) to 4th and 8th grade students every four years are carried out in order to determine the place of countries at the international level, the differences in education systems and the trend in student achievement. Results of PISA and TIMSS tests analyze the current state of their educational research, change in the curriculum to increase student success, lead to studies on teacher competencies. In these tests, student achievement and tendencies in mathematics, which are considered important for the development of countries, are not at the desired level. Since student achievement and tendencies in mathematics, which are considered important for the development of countries in these tests, are not at the desired level, many studies are carried out to determine the teacher competencies in mathematics education. As Shulman (1986, 1987) stated, studies on teacher knowledge that reveal a teacher’s capacity or competence reveal the inadequacy of teacher candidates and teachers in effective mathematics teaching (Bahar, 2019; Gökce, 2019; Gökkurt, Şahin, Soylu & Doğan, 2015; Kutlu, 2018; Kutluk, 2011; Murtafiah & Lukitasari, 2019; Yeşildere & Akkoç, 2010; Türnüklü & Yeşildere, 2007).

Considering that mathematics is an important discipline in the development of countries, it is necessary to have an idea to evaluate the knowledge of mathematics teachers who are the implementers of the mathematics curriculum. In this study, theoretical studies which aim to identify, explain and develop the categories of teacher knowledge from the perspective of mathematics education are evaluated.

## Teacher Knowledge in Mathematics Education

Shulman’s (1986, 1987) studies on teacher knowledge based on “knowing and teaching” perspective directed teacher education researchers to study the categories and sub-categories of this knowledge that are necessary for effective teaching. One of the studies conducted for this purpose was carried out by Ball, Thames and Phelps (2008) in the framework of mathematics education.

Ball, Thames and Phelps (2008) categorized the fields of knowledge which is required to teach mathematics as subject content knowledge and pedagogical content knowledge as shown in Figure 1.

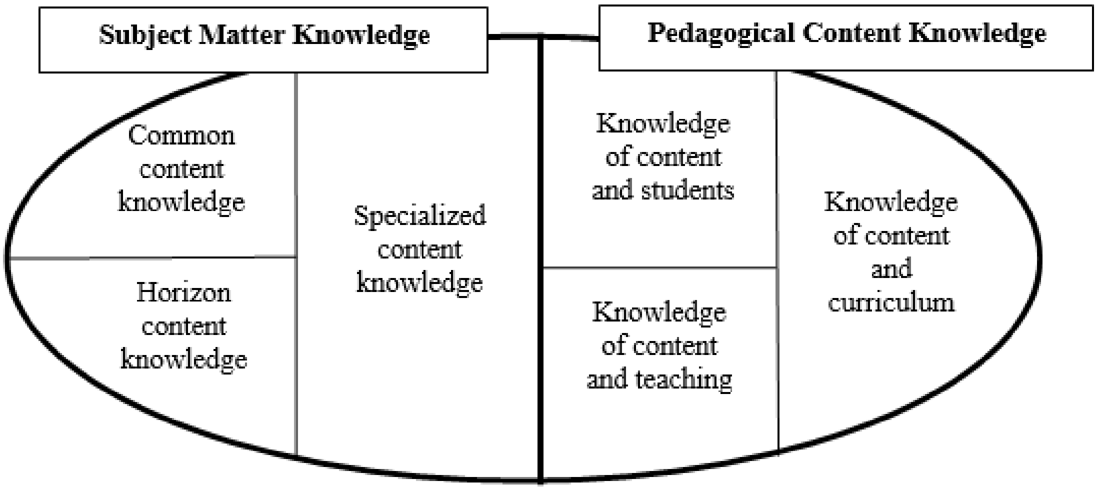


Figure 1. Comparison of Shulman's (1986) and Ball, Thames and Phelps (2008) Knowledge Categories (Ball, Thames & Phelps, 2008, p.403)

Subject matter knowledge is defined as the mathematical knowledge to teach needed mathematics content. It is divided into sub-categories such as general content knowledge, specialized content knowledge and horizon content knowledge.

It is the mathematical knowledge and skills that all well-educated adult should have. For example, the knowledge and skills required to be done in the correct form of the243x15 transaction are included in the general content knowledge category.This information dimension is to be able to identify the mistakes students make, to be able to recognize inappropriate definitions; It provides speaking and writing by using correct terms and notations.

Horizon content knowledge covers the knowledge and awareness of how the mathematics subjects in the curriculum are interconnected. For example; the information about which topics a mathematical concept, transaction or subject is basic or preliminary information are within the scope of this knowledge dimension. Private content knowledge is the knowledge required to teach mathematics beyond what is expected of well-educated adults. Special content knowledge is the knowledge which is necessary to teach mathematics beyond what is expected of well-educated adults. For example; the type of information required to make instructional explanations in accordance with the algorithm behind the $5\ \frac{1}{2}$ transaction is included in this scope.

Unlike pedagogical content knowledge, Mathematical special content knowledge covers the mathematical knowledge required to teach mathematics. This type of knowledge required for the teaching of a mathematical task is generally based on information such as making appropriate mathematical explanations, using representations, justifying and relating.

This type of knowledge, which is necessary to teach a mathematical task, is generally based on information such as making appropriate mathematical explanations, using representations, justifying and relating. Presentation and representation of mathematical ideas, with the mathematical ideas underlying the representations or other representations, providing mathematical examples suitable for critical points, making explanations for reasons, relating a topic to learned topics or topics to be learned, choosing and developing useful mathematical definitions, adapting and organizing the content, using the mathematical language and notation appropriately are seen as knowledge and skills within the scope of special content knowledge. (Ball, Thames&Phelps, 2008).It can be said that Ball, Thames and Phelps require a strong general content knowledge and support pedagogical content knowledge when it comes to the definition of special content knowledge and the skills it contains. In addition, defining specific content knowledge skills related to mathematical tasks can help mathematics teachers question their own actions and strategies and develop course content.

Ball, Thames, and Phelps (2008) categorize the subcomponents of pedagogical content knowledge, which is another dimension of teacher knowledge, as student knowledge related to the content, content teaching knowledge and curriculum knowledge. This categorization differs according to the PAB categories of Shulman (1987).

While Shulman (1987) considers curriculum knowledge as a different dimension of knowledge from pedagogical content knowledge; Ball, Thames, and Phelps (2008) consider curriculum knowledge as a subcomponent of pedagogical content knowledge. The curriculum affects the scope and presentation of the presented content. Considering that pedagogical content knowledge is related to a specific form of content knowledge (Shulman, 1986) that represents the teachable aspects of content, it is clear that curriculum knowledge will also affect pedagogical content knowledge. For this reason, it can be said that Ball, Thames and Phelps(2008), who dealt with curriculum knowledge as a sub-component of pedagogical content knowledge, made an appropriate determination.

In the literature, there are also different categorization of mathematics-specific content knowledge, which Shulman (1986) and Ball, Thames and Phelps (2008) express as subject matter knowledge. For example, Skemp (1976) embraces the subcomponents of mathematics content knowledge as conceptual and operational knowledge. Conceptual knowledge; It is the knowledge of mathematical concepts and the mutual transitions and relations between concepts (Skemp, 1976). Moreover conceptual knowledge covers the mathematical meaning behind rules, relationships, generalizations and operations. Operational knowledge is the knowledge of mathematical methods, rules, and algorithms (Skemp, 1976).

Another categorization of the content knowledge was made by Ball, Lubienski, and

Mewborn (2001). According to Ball, Lubienski and Mewborn, mathematics content knowledge has two sub-components. These are mathematics knowledge and mathematics-related knowledge. According to this categorization mathematics knowledge covers operations and core mathematical meanings. The knowledge based on mathematics covers mathematical presentations and knowledge about how mathematics has developed and changed as a discipline (Ball and others., 2001).

In the teacher education literature, pedagogical content knowledge was first introduced as a category of knowledge by Shulman (1986). Shulman's studies (1986, 1987) accelerated the studies about pedagogical content knowledge. The number of studies on what this knowledge is and its subcategories has increased (Ball et al., 2008; Rowland, Huckstep, & Thwaites, 2003; Rowland, 2005; Rowland, 2013).There have also been studies examining pedagogical content knowledge in terms of mathematics education. Baki (2010) carried out one of these researches and embraces with pedagogical content knowledge as teaching knowledge. According to Baki (2010), the skills required by the mathematics teacher knowledge dimension are expressed as follows (p.24)

- Knowing what to teach in the curriculum
- Knowing and relating the learning areas of the curriculum
- Knowing the achievements of sub-learning areas
- Knowing how the learner comprehend
- Know the student's current operational and conceptual knowledge specific to the subject
- Knowing subject-specific special teaching methods
- To be able to design subject-specific material
- To be able to organize subject-specific learning activities
- Assessment and evaluating student's learning

It is seen that the knowing the curriculum to be taught from these expressed skills, knowing and relating to the learning areas of the curriculum, and the skills of determining the acquisitions of sub-learning areas are related to the curriculum knowledge from the PCK components. (Ball et al, 2008; Shulman, 1986, 1987). The ability to know how the student understands and the subject-specific operational and conceptual knowledge points to the students' understanding knowledge, one of the PCK components expressed in the literature (Ball et al., 2008; Shulman 1986). Skills about knowing how the student understands and current subject-specific operational and conceptual knowledge points to the students' understanding knowledge, which is one of the PCK components expressed in the literature. (Ball et all., 2008; Shulman 1986). Knowing subject-specific teaching methods, designing materials, organizing learning activities, and assessment and evaluating student learning are the knowledge about teaching knowledge from the components of PCK (Ball et al., 2008; Shulman, 1986). Baki (2010) tried to explain the components of mathematics teaching knowledge with a diagram as in Figure 2 with the help of this framework. When Figure 2 is examined, Shulman's (1986, 1987) student knowledge is considered as Ball, Thames and Phelps' (2008) the knowledge of content and students, whereas Baki (2010) considers it as student's current mathematics knowledge.  Knowledge of teaching strategies revealed by Shulman (1986, 1987), is embraced in the form of presentation of the subject and special teaching methods and strategies by Baki (2010). The component that Baki (2010) discussed as "the place of the subject in the mathematics curriculum and its relation with other subjects"; it is discussed as a separate category of knowledge by Shulman (1987), and curriculum information as a sub-knowledge category of PAB by Ball, Thames and Phelps (2008). The last component put forward by Baki (2020) can be associated with Shulman's (1987) category of knowledge about educational purposes and values.

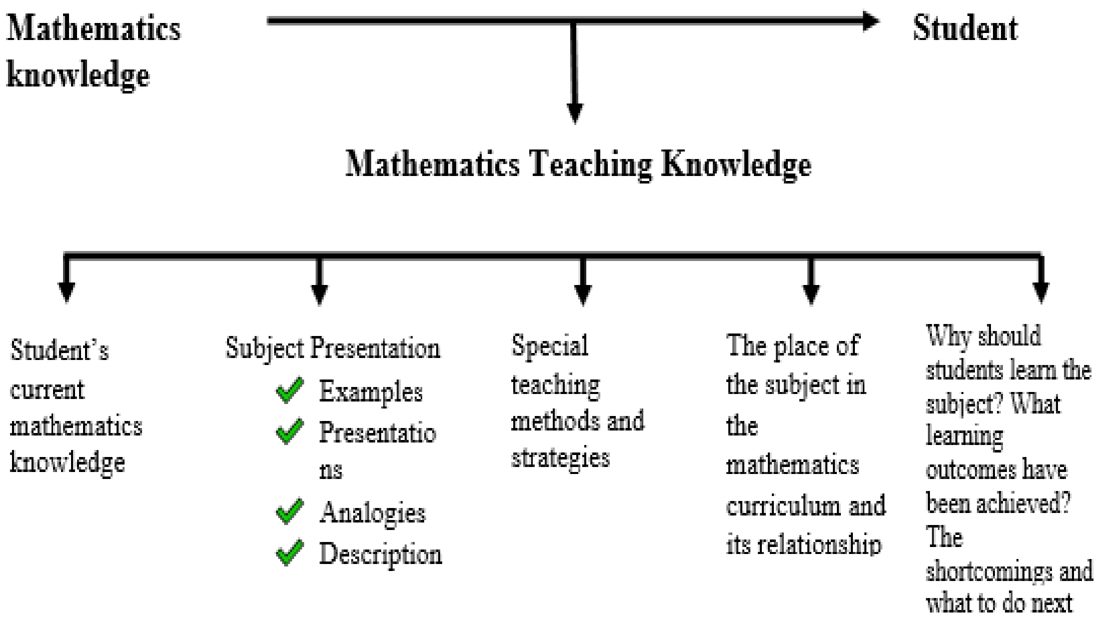


Figure 2. Components of Mathematics Teaching Knowledge (Baki, 2010, p.25)

## Evaluation of Teacher Knowledge in Mathematics Education

While categorizing teacher knowledge in the literature of mathematics education, it is known that evaluation studies have been conducted. (Kinach, 2002; Rowland, Huckstep ve Thwaites, 2003; Rowland, 2005; Rowland, 2013). Studies on teacher knowledge emphasize that strong pedagogical content knowledge requires deep field knowledge. (Shulman, 1986; Ball, Thames&Phelps, 2008). Kinach (2002) stated that instructional explanations, which are an indicator of pedagogical content knowledge, reveal teachers' understanding of the content knowledge they have and tried to define the levels of this understanding. Kinach (2002) has developed a framework based on evaluating the instructional explanations made by the teachers from the perspective of mathematical content and pedagogical content knowledge, based on Skemp (1976) and Perkins and Simmon's (1987) categorization for mathematical content knowledge. The framework that Kinach (2002) put forward to evaluate the knowledge of mathematics teachers is shown in Table 1.

Table 1: Understanding Levels Developed by Kinach (2002)

| | |
|---|---|
| Instrumental understanding | Content-levelunderstanding |
| Relational understanding | Theconceptlevel of disciplinaryunderstanding |
| | Problem-solvinglevelunderstanding |
| | Epistemic-levelunderstanding |
| | Inquiry-levelunderstanding |

According to Kinach (2002), the indicator of operational understanding is called content level understanding.Content level understanding of mathematics includes rules, operations and basic methods specific to mathematics. On the other hand, relational understanding is observed at four levels. The first of these levels is the concept level of disciplinary understanding which covers defining patterns and relationships, knowledge and experiences based on its classification and generalization. Problem-solving level understanding covers schemes based on the use of analytical tools and methods in solving problems specific to mathematics. Epistemic-level understanding is based on verifying and proving mathematical thoughts and justifying explanations. The framework put forward by Kinach (2002) focuses on evaluating the teacher knowledge on the basis of the quality of the instructional explanations

In the literature, it has been seen that there are studies which aim to evaluate the teacher knowledge according to the characteristics shown in the whole teaching process. (Rowland, Huckstep&Thwaites, 2003; Rowland, 2005; Rowland, 2013). According to Rowland (2005), who conducts studies aimed at defining and developing a theoretical framework to describe and analyze prospective teachers' mathematical knowledge thoroughly, such characteristics of teachers can be evaluated in the best way while teaching, in other words, in practice. This theory, called "The Knowledge Quartet", was first developed at Cambridge University between 2002-2004 (Rowland, 2013). According to this understanding shaped by a series of researches and studies, there are four dimensions of knowledge that a teacher should have (Rowland, Huckstep& Thwaites, 2003).

- Foundation

- Transformation
- Connection
- Contingency

The basic knowledge category consists of the knowledge, beliefs and understandings that prospective mathematics teachers have gained in academy and prepare them for their future roles. Mathematics knowledge and understanding, understanding and beliefs based on thoughts resulting with questioning in learning and teaching mathematics are classified as key components of this theoretical background (Rowland, Huckstep&Thwaites, 2003; Rowland, 2013).

Action knowledge represented by planning for teaching and instruction itself constitute the transformation category which is the second category.There is the ability to transform content knowledge into strong pedagogical forms which was expressed by Shulman (1987) at the center of this category. From this perspective; the choices and examples used in the teaching of the course are important in terms of creating a conceptual background in mathematics teaching, helping to show the language acquisition and mathematical processes. (Rowland, Huckstep & Thwaites, 2003; Rowland, 2005; Rowland, 2013)

Establishing connection knowledge category; it is about consistency in planning and teaching the lesson parts and a set of lessons. The consistency here is the choices that reflect knowledge of their structural connections in mathematics and covers the ordering of the lesson parts with the directions, as well as the awareness of the cognitive demands of different mathematical topics and tasks (Rowland, Huckstep & Thwaites, 2003; Rowland, 2005; Rowland, 2013).

In the last category, the ability to "think for someone else" is at the forefront. And It is about events that are almost impossible to plan, and means being prepared for student ideas and being able to direct them appropriately Rowland, Huckstep & Thwaites, 2003; Rowland, 2005; Rowland, 2013). In the constructivist approach, it is taken into account that the student's contributions in the course constitute an important perspective in teaching. In this respect, the last category is very important as it includes students' possible questions and thoughts in the planning and teaching of the lesson.Rowland (2013) classified the indicators about these categories of information as in Table 2.

It handles the mathematics knowledge and mathematics teaching knowledge of the Knowledge Quartet Modeltogether. This indicates that both types of knowledge are important. Considering that pedagogical content knowledge covers blended aspects of content and pedagogical knowledge; a good mathematics pedagogical content knowledge requires a strong mathematics subject matter knowledge. Although good math subject knowledge is a must for pedagogical content knowledge, it does not guarantee that good. In this respect, Category codes of the Knowledge Quartet Modelshows that it can be said that it is a good synthesis of mathematics subject knowledge and pedagogical content knowledge. In addition, it emphasizes that a teacher's mathematics subject knowledge and pedagogical content knowledge can be observed in practice in the best way. The fact that the category codes are application-oriented also shows the usefulness of the Knowledge Quartet Model.

Table 2. The Knowledge Quartet's Dimensions

| Knowledge Dimension | |
|---|---|
| Foundation | Adherence to text book<br>Awareness of purpose<br>Concentration on procedures<br>Identifying pupil errors<br>Overt display of subject knowledge<br>Theoretical under pinning of pedagogy<br>Use of mathematical terminology |
| Transformation | Choice of examples<br>Choice of representations<br>Teacher demonstration<br>Use of instructional materials |
| Connection | Anticipation of complexity<br>Decisions about sequencing<br>Making connections between concepts<br>Making connections between procedures<br>Recognition of conceptual appropriateness |
| Contingency | Deviation from lesson agenda<br>Responding to students' ideas<br>Responding to the (un)availability of tools and resources<br>Teacher in sight |

## Conclusion

In this study, it is aimed to present a perspective based on the researches on defining, classifying and evaluating teacher knowledge from the mathematics education point of view. It is seen that the studies conducted try to determine the dimensions of teacher knowledge and the interaction among these dimensions of knowledge. Among these dimensions of knowledge, mostly content knowledge and pedagogical content knowledge, the subcategories of these dimensions of knowledge and the interaction among them were emphasized. (Baki, 2010; Ball, Lubienski & Mewborn, 2001; Ball, Thames & Phelps, 2008; Kinach, 2002; Rowland, Huckstep & Thwaites, 2003; Rowland, 2005; Rowland, 2013; Shulman 1986, 1987; Skemp, 1976)

More relational knowledge types in the dimension of mathematics-specific content knowledge of teacher knowledge is seen as the necessity of a conceptual and deep mathematical knowledge (Skemp, 1976; Kinach, 2002). Strong content knowledge is a prerequisite for an effective teaching process; but it is not enough. The knowledge and skills that can transform the content knowledge into pedagogically strong forms should be possessed (Shulman, 1987).

Student knowledge and instructional strategy information come to the fore in the sub-dimensions of pedagogical content knowledge (Baki, 2010; Ball, Thames & Phelps, 2008; Shulman 1986, 1987). Besides the categorization of teacher knowledge related to mathematics, studies aimed at analyzing and evaluating teacher knowledge also shed light on teaching skills that need to be developed and set application-based evaluation criteria (Kinach, 2002; Rowland, Huckstep, & Thwaites, 2003; Rowland, 2005; Rowland, 2013). The contingency dimension in The Quartet Knowledge model is an important detail among these application-based frameworks. It is emphasiezed that being prepared for problems that may occur in the classroom and the capacity to turn these into an educational opportunity is a necessary teacher competence.

It is important to define and elaborate the knowledge and skills of teachers who are implementers of teaching programs in advancing mathematics education, and to create evaluation frameworks, in terms of shedding light on teacher training programs. Accordingly, teacher training programs and in-service training programs for teachers working currently are required to improve instructional knowledge and skills.

## References

Akkoç, H.,& Yeşildere, S. (2010). Investigating development of pre-service elementary mathematics teachers' pedagogical content knowledge through a school practicum course. *Procedia-Social and Behavioral Sciences*, *2*(2), 1410-1415.

Bahar, G., (2019). *Measuring pre-service teachers' mathematics content knowledge and mathematics pedagogical content knowledge on ratio and proportion.* Unpublished master thesis, Boğaziçi University, Institute for Graduate Studies in Science and Engineering, İstanbul

Baki, M. (2010). An evaluation of teacher training from undergraduate and postgraduate dimensions. *Journal of Inonu University Faculty of Education, 11*(3), 15-31.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. *Handbook of research on teaching,* 4, 433-456.

Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching what makes it special?. *Journal of Teacher Education*, *59*(5), 389-407.

Gökce, R., (2019). *Examining middle school mathematics teachers' content and pedagogical content knowledge of statistical reasoning.* Unpublished doctoral dissertation, Pamukkale University, Institute of Educational Sciences, Denizli

Gökkurt, B., Şahin, Ö., Soylu, Y. ve Doğan, Y. (2015). Pre-service teachers' pedagogical content knowledge regarding student mistakes on the subject of geometric shapes. *Elementary Education Online, 14*(1). Retrieved fromhttp://ilkogretim-online.org.tr/

Kinach, B. M., (2002). Understanding and learning-to-explain by representing mathematics: Epistemological dilemmas facing teacher educators in the Secondary mathematics "methods" course. *Journal of Mathematics Teacher Education, 5*, 153–186.

Kutlu, D. (2018). *Examination of the Pedagogical Content Knowledge Levels of Novice Secondary School Mathematics Teachers.* Unpublished master degree thesis, Karadeniz Technical University, Institute of Educational Sciences, Trabzon.

Kutluk, B. (2011). *The investigation of elementary mathematics teachers knowledge of student difficulties related to pattern concept.* Unpublished master degree thesis, Dokuz Eylul University, Institute of Educational Sciences, İzmir

Murtafiah, W.,& Lukitasari, M. (2019). Developing Pedagogical Content Knowledge of Mathematics Pre-Service Teacher through Microteaching Lesson Study. *Online Submission*, *13*(2), 201-218.

Perkins, D.N. & Simmons, R. (1988). Patterns of misunderstanding: An integrative modelfor science, math, and programming. *Review of Educational Research*, 58(3), 303–326.

Rowland, T., Huckstep, P., & Thwaites, A. (2003). The knowledge quartet. *Proceedings of the British Society for Research into Learning Mathematics*,*23*(3), 97-102.

Rowland, T. (2005). The knowledge quartet: A tool for developing mathematics teaching. In A. Gagatsis (Ed), *Proceedings of the Fourth Mediterranean Conference on Mathematics Education* (1st ed., pp. 69-81) Nicosia, Cyprus: Cyprus Mathematical Society.

Rowland, T. (2013). The knowledge quartet: The genesis and application of a framework for analysing mathematics teaching and deepening teachers' mathematics knowledge. *Sisyphus-Journal of Education*, *1*(3), 15-43.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Skemp, R.R (1976) Relational Understanding and Instrumental Understanding, *Mathematics Teaching, 77*, 20-26.

Türnüklü, E. ve Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Preservice primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1* (October). Retrieved fromhttp://www.k-12prep.math.ttu.edu