Models of the Mathematical Curriculum for the VI Middle School Grade Developed in B&H, Croatia, Montenegro and Serbia

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Introduction

The term 'Mathematics Curriculum' refers to the lessons and academic content taught in a school in a specific grade. In dictionaries, curriculum is often defined as the courses offered by a school, but it is rarely used in such a general sense in schools. Depending on how broadly educators define or employ the term, curriculum typically refers to the knowledge and skills students are expected to learn, which proficiencies educated student will develop (Kilpatrick, Swafford and Findell, 2001). It includes the learning standards or learning objectives they are expected to meet (Bloom, 1956; Krathwohl, Bloom and Masia, 1964). In addition, it conclude the units and lessons that teachers teach; the assignments and projects given to students; the books, materials, videos, presentations, and readings used in a course; and the tests, assessments, and other methods used to evaluate student learning. Since a mathematics curriculum of any Middle school grade is one of the foundational elements of effective schooling and mathematics teaching, it is often the object of reforms, most of which are broadly intended to either mandate or encourage greater curricular standardization and consistency across states, schools, grade levels, included subject areas, and so on...

Teachers typically modify what they teach and bring their curriculum into "alignment" with the learning expectations outlined in the standard, in the case of a socio-political community adopting new learning standards in mathematics at state, district or school levels. While the technical alignment of the curriculum with the standards does not necessarily mean that teachers are teaching in accordance with the standards - or, more to the point, that students actually achieve those learning expectations-the standards of learning remain a mechanism by which policy makers and school managements attempt to improve curriculum and teaching quality. The 'Common Core of National Standards for Mathematics', if it existed, for example, could be interpreted as a national and academic effort to influence curriculum design and teaching quality in schools through the adoption of new learning standards by the Ministry of Education.

Another reform strategy that indirectly influences curriculum is assessment, since the methods used to measure student learning compel teachers to teach the content and skills that will eventually be evaluated. The most commonly discussed examples are standardized testing and high-stakes testing, which can give rise to a phenomenon informally called "teaching for the test."

Schools may try to improve curriculum quality by bringing teaching activities and course expectations into "alignment" with learning standards and other school courses - a practice sometimes called "curriculum mapping." The basic idea is to create a more consistent and coherent academic program by ensuring that teachers teach the most important content and eliminate learning gap that may exist between sequential courses and grade levels. For example, teachers may review their mathematics program to ensure that students are actually be educated in every arithmetical part (when students are expected to develop arithmetic and early-algebraic thoughts) or geometrical parts (when students are expected to understand and accept the planned elements of geometry at 'Level 0' and 'Level 1' according to the van Hiele classification (Van Hiele, 1986)), offered in the school, not only reflects the expected learning standards for that, but also that it also prepares students for algebraic and geometrical parts in V - VIII grades (when elements of algebraic thinking are expected to develop in cognitive levels of educated students, and when students should understand and accept geometric concepts and processes with them at 'level 1' and within 'level 2').

The design and goals of any mathematics curriculum reflect the educational philosophy-whether intentionally or unintentionally-of the curriculum designers who developed it. Consequently, the curriculum reform may occur through the adoption of a different philosophy or model of teaching by a school or educator. Schools that follow the 'The traditional approach' or 'Model of Didactic situations' (Brousseau, 2002; Sierpinska, 2003) or 'Theory of Realistic Mathematics Education' (Freudenthal, 1991; Gravemeijer, 1994), for example, embrace a variety of approaches to teaching generally known as project-modeling based learning, which encompasses related strategies such as community-based learning and authentic learning. Also, under the philosophy incorporated into the curriculum, we mean the orientation of a legislator of a socio-political community and curriculum designers to accept and encourage a differentiated approach to teaching and students' learning.

In the school systems of socio-political communities whose mathematical education we observe, they have a centralized system of curriculum design by their ministry of education. At the beginning of his career, a teacher of mathematics asks himself, 'What should a mathematical curriculum contain?' Such a teacher needs of any kind of help that would enable him to organize his teaching more efficiently and that the results of his work would be to reach the goals of the teaching of mathematics in a high percentage.

He needs not only the goals of teaching mathematics and planned outcomes, but also much more important instruction with many details on how to achieve these goals and what teaching tasks to accomplish with the intention to achieve these goals. In addition, he needs the detailed instruction which teaching tasks he should be realized in order to achieve the affective goals (Grootenboer, Lomas and Ingram, 2008) of the teaching planned for the appropriate level in the teaching process. Then, he needs instructions on how to meet expectations of the social and academic communities in the teaching process and with which success he has achieved them. For example, when it comes to socio-mathematical norms, he needs clear instructions how to check that students have adopted some of these planned norms. Also, if in a mathematics curriculum for a Middle School grade there is the orientation 'teacher must develop the students' ability of logical thinking', the teacher needs detailed instructions on which elements of logical thinking this applies. Has the designer of the curriculum used the slogan 'logical thinking' in rhetorical and colloquial sense, or was he thinking of some specific logical tautologies and rules of concluding?

The topic of this research is of general interest in the international community of researchers in mathematics education (Robitalle and Dirks, 1982; Hawson, 1983; MEPRC, 2003; Anderson, 2009; Yee, 2010; Gürlen, 2015; Seah et al, 2016; Hayes, 2017). Over the past twenty years, this academic community has shown a growing interest in comparative studies on mathematical curricula of various socio-political communities (For example, Seah et al, 2014; Hasić and Romano, 2018).

In this article, comparing the curricula of mathematics for the VI grade of Middle Scholl in B&B, Croatia, Montenegro and Serbia, we will estimate the characteristics of these models. This will apply to both 'teaching plans' and 'teaching programs' elements mentioned socio-political communities.

Comparison of the Curriculum models

Our intention with this text is to open a dialogue between the designers of mathematical syllabus. In the school systems of B&H, Croatia, Montenegro and Serbia, the term 'teaching plan' refers to the planned annual number of hours of mathematics. The term 'teaching program' refers to the objectives of teaching mathematics (general and individual), teaching contents, planned outcomes of mathematics and didactic instruction for teaching realizes. In B&H and Montenegro, the basic education system (Primary grades and Middle school's grades) lasts nine years. Therefore, the corresponding classes are: VII (B&H), VI (Croatia), VII (Montenegro) and VI (Serbia). Information on mathematics teaching plans for the observed grade in Middle schools in mentioned countries is presented in the following table:

Table 1. Teaching Plans of Mathematics for VI Grade

State	Number of Weekly Classes	Number of Annually Classes	FB&H = 1	Grades
B&H [FB&H]	4	140	1.000	II
B&H [RS]	4	144	1.029	VII
Croatia	4	140	1.000	VI
Montenegro	4	136	0.971	VII
Serbia	4	144	1.029	VI

The teaching contents of mathematics for the observed grade are the following

Table 2: Teaching Program of Mathematics for VI Grade

Thematic content	B&H		Croatia	Montenegro	Serbia
	RS	FB&H			
Fractions in decimal form					
Integers	+(25)		+	+(42)	+(24)
Rational numbers	+(45)	+(15)	+	+(41)	+(45)
Angle and Triangle		+(25)	+	+(23)	+(30)
Quadrilateral	+(22)	+(30)	+	+(20)	+(20)
The length of triangular line and	+	+(30)			
The length of quadrilateral line	+(18)	+(30)	+		
Areas of triangles and quadrilaterals	+				+(17)
Elemental combinatorial tasks				+(6)	

Legend: Numbers in brackets indicate the suggestions of the Ministry of Education about the necessary hours for teaching and learning of noted themes

[FB&H] In an official document from the Ministry of Education, Central Bosnia Canton, the goals of teaching mathematics are cumulatively for all grades of the Middle School. They are also classified into two categories: Goals of Course and Tasks of Course. The first cluster refers to the mathematical content of courses in all grades of the Middle School. The second cluster consists of nine statements of general character. Professionally speaking, an academic person well educated in the domain 'Didactic of Mathematics', none of the above statements can hardly classify them into one of the categories of teaching goals of mathematics according to Bloom's classification.

[RS] In the official document of the Ministry of Education of the Republic of Srpska, the objectives of teaching mathematics for the 7th grade of the Middle School are classified into two categories: General goals of teaching Mathematics and Specific goals of teaching mathematics. These are mostly rhetorical and plausible statements in general forms. In the second cluster, there are listed the planned outcomes of teaching mathematics in the 7th grade of the Middle School. Each mathematics teacher, familiar with Bloom's classification of teaching goals, finds it very difficult to recognize them as some of the Cognitive and / or More Effective Mathematics teaching goals

[MNE] The aims of teaching mathematics in the Montenegrin Middle School are realized

through the realization and achievement of cognitive and process goals. Cognitive objectives include the knowledge that the student will acquire through the adoption of mathematical contents given in the programs. Process goals include skills and values that are evolving during and during the learning process. Although the term 'process goals' of teaching mathematics is not close to us, in order to illustrate of what is meant by such goals in the official document of the Institute for Education of Montenegro, we instance the following: Through the 'process goals' students should develop:

- Possibility of logical thinking, conclusion and generalization and mathematical proving;

- Quality and ability to formulate problems;

- Possibility of problem solving;

- The skills of interpreting the data shown in diagrams, tables or charts of different species;

- The use of geometric accessories and measuring instruments;

- Possibility to recognize situations in everyday life in which mathematical knowledge can be applied;

- Innovation and creative thinking;

- Possibility of critical thinking;

- Cultural, ethical, aesthetic and work habits, criteria and abilities.

[CRO] The goal of teaching mathematics is to acquire basic mathematical knowledge necessary for understanding phenomena and laws in nature and society. Also, the goal is to acquire basic mathematical literacy and the development of abilities and skills in solving mathematical problems.

[SRB] The goal of teaching mathematics in Elementary school is to ensure that all pupils acquire basic linguistic and mathematical literacy and advance to the realization of appropriate standards of educational achievement, and that:

- Enable students to solve problems and tasks in new and unknown situations;

- Enable students to express and explain their thinking and discuss with others;

- Develop motivation for learning and interest in the subject contents;

- Ensure that learners adopt the elementary mathematical knowledge required for understanding of phenomena and laws in nature and society;

- Enable students to apply mathematical knowledge in solving diverse tasks from life practice;

- Represents the basis for successful continuation of mathematics education and for self-education;

- Contributes to the development of mental abilities, forms a scientific view of the world and

- Comprehensive development of students' personality.

Our finding

Looking at the of mathematics curricula models for the VI year of the Middle School B & H, Croatia, Montenegro and Serbia, we are inclined to believe that there is general social orientation in the observed socio-political communities that the teaching and learning of mathematics is one of the key foundations of education. The explanation for social orientation for teaching Mathematics as a fundamental subject at the Middle School levels is presented below:

- Mathematics is a powerful means in a technology-oriented and information-rich society to help students acquire the ability to communicate, explore, conjecture reason logically and solve problems using a variety of methods.

- Mathematics provides a means to acquire, organize and apply information, and plays an important role in communicating ideas through pictorial, graphical, symbolic, descriptive and analytical representations. Therefore, mathematics at the middle school level helps to lay a strong foundation for both continuing education in secondary schools and students' lifelong learning, and provides a platform for the acquisition of new knowledge in this rapidly changing world. Therefore, mathematical experiences acquired at the Middle school level enable students to become mathematically literate citizens who are more able to cope with the demands of their future everyday life. The mastered mathematical proficiencies are tools to help students enhance their understanding of the world. They provide a foundation for the study of other disciplines in secondary and post-secondary education.

- Mathematics education is an intellectual endeavor through which students can develop their imagination, initiative, creativity and flexibility of mind, as well as their ability to appreciate the beauty of nature. Mathematics is a discipline which plays one of central roles in human culture. The development of mathematical thinking (arithmetic, early-algebraic, algebraic and geometric) in students greatly enhances their chances for better progress in future schooling and day-to-day challenges in forthcoming working responsibilities. We deeply believe that quality mathematical education and completely developed mathematical proficiencies, especially in. Affective domains, are allow students to become more moral persons and honest and better people.

The goals of the observed models of the mathematics curriculum, although not explicitly pointed out, have to develop in students:

(a) The ability to think critically and creatively, to conceptualize, inquire and reason mathematically, and to use mathematics to formulate and solve problems in everyday life, as well as in mathematical contexts and other disciplines;

(b) The ability to communicate with others and express their views clearly and logically in mathematical language;

(c) The ability to understand and manipulate numbers (capability to think arithmetically), symbols (capability to think algebraically) and other mathematical objects (capability to think geometrically);

(d) Number sense, symbol sense, geometric and spatial sense, measurement sense and capacity to appreciate structures and patterns;

(e) A positive attitude towards the learning of mathematics and an appreciation of the aesthetic nature and cultural aspects of mathematics.

Comments and Observations

A natural question posed by the authors of this research is the following: Are mathematics teachers the ones for whom such designed models of curriculum mathematics for the VI grade of Middle School are intended? Are these designed curricula enabled working teachers to construct with high standards their global and executive plans for the realization of their teaching? According to our deep conviction, before work, there is still a lot of work in the design, preparation and construction of executive plans for the implementation of these models. Curriculum models thus designed are much more relevant to school administration and supervisory services from Pedagogical Institutes than to implementers in the classroom. The problems encountered by the implementers of mathematics teaching in this Middle School class are the same as the problems that other teachers have. We do not know whether there are researchers in the observed socio-political communities whose field of interest is the development of mathematical curricula. The usual practice in these communities is that their Pedagogical Institutions form an ad hoc commission composed of teachers of mathematics who design curriculum models. It has been shown that such an approach is insufficient in quality and does not bring quality improvement.

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