Mathematics (M) in STEM

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Nowadays STEM is one of the hot topics in our world. On the one hand most of the readers know that STEM stands for science, technology, engineering, and mathematics, on the another hand they probably agree on the equally importance of each subject for human beings. However for some of us (i.e. mathematics educators), one letter, which is M, is more salient than others. Especially in STEM education "what is the meaning of mathematics?" is a crucial question, which will be discussed in the rest of this chapter.

Definitions of M in STEM could have different meanings for individuals. Clark-Wilson and Ahmed (2009) stated this difference as how M in STEM should be interpreted; it could be increasing the achievement in mathematics or including mathematics in an integrated curriculum. The former allows us to segregate mathematics but the latter requires interwoven structure, which would be more proper to indicate as M in STEM. Therefore, it would be fair to state that mathematics in STEM should be used more than a service subject for the other three subjects (Clark-Wilson & Ahmed, 2009).

Mathematics could be accepted as an essential subject for many, and for mathematics teachers it is serious and valuable subject because it provides a connection between subjects. However mathematics teachers had some concerns about understanding the significance of "mathematics" by others. In fact, Coad (2016) reflected teachers' opinions in his paper where he gathered and presented their ideas emerged from a meeting (i.e., workshop). These reflections were as follows: 1) to use mathematics as a data presentation tool can result in discrediting mathematics; 2) do not expect students to understand every detailed mathematical procedure in STEM activities yet accept mathematics as a part of STEM projects; 3) mathematics is inevitable component of STEM activities; 4) assessing mathematical achievement and engagement is important; 5) the difficulty of differentiation of mathematical outcome in STEM activities. Also Clark-Wilson and Ahmed (2009) focused on determining mathematics teachers' practices, who involved in STEM collaborative professional development (CPD) projects, by examining their perceptions about M in STEM and exploring what makes rich STEM CPD and how mathematics was defined in STEM. The results were that there should be definitely a shared vision and pedagogy about mathematics in STEM. In addition to that developing the view of using mathematics to a broader perspective was another result. One of the concerns was also giving each subject an essential importance and keeping their integrity as well. Difficulty of preparing authentic mathematical activities without focusing only about the content with concerns was another problem. These reflections and results revealed that mathematics' role in STEM was not crystal clear, which is understandable because I believe mathematics' role in STEM is not just about only being a subject it also has different roles. Even though mathematics as an academic subject is not included in a STEM activity, this does not mean it is not included because from my standpoint M in STEM also means "mathematical thinking and understanding", "mathematical problem solving", "mathematical reasoning", and " mathematical modelling".

Mathematical Thinking and Understanding

Instead of teachers teaching students any rule about any subject with rote memorization, teaching them a style of thinking is in need of today's world (Winchester, 2007). Mathematical thinking is a style of thinking that is an interrogation of our world (Winchester, 2007), and therefore it is used widely in educational context. For instance, a teacher, who searched for the definition of mathematical thinking to ensure that she conveys thinking mathematically to her students, realized that the symptoms in mathematical thinking were actually "generic thinking skills and could be applied to all subjects and problems that we encounter in life" (Pitt, 2002, p. 4). This is exactly what educators are looking for in STEM projects- thinking skills required to solve real-life problems. Therefore, mathematical thinking is one of the reasons why "M" needs to be considered as an essential element of STEM.

There is not a specific and one definition of mathematical thinking (Lane, 2005; Sternberg, 1996). Researchers focused on analyzing, conjecturing, proving, reasoning, justifying, formalizing, generalizing, and advanced thinking (Ball, 2002; Dreyfus, 1990; Lane, 2005; Mason, Burton & Stacey, 2010; Polya, 1954; Selden & Selden, 2005). Sternberg (1996) addressed different explanations of mathematical thinking made by researchers, organized all approaches, and categorized these approaches as psychometric, computational, anthropological, pedagogical, and mathematical. Sternberg (1996) pointed out following abilities as mathematical thinking according to different approaches: Fluid intelligence (i.e., the importance of sequence and speed or reasoning), crystallized intelligence (i.e., the importance of knowledge and language skills), memory ability, visual apprehension (Carroll, 1996), information-processing (i.e. quantitative & qualitative reasoning) (Mayer & Hegarty, 1996), analogical thinking (Ben-Zeev, 1996), and creative thinking. Especially Ben-Zeev's (1996) explanation of mathematical thinking was dramatic because analogical thinking in mathematics occurs when "one forms a mapping between past problems one has solve and the present problem one is seeing to solve, and also when one seeks to see the relations among a set of problems one needs to solve in the present" (Sternberg, 1996, p. 307).

This explanation shows us that this style of thinking is actually what commonly used in STEM projects or challenges. Starting from this point of view, it would be unfair not to state the mathematical thinking as a part of STEM.

Mathematical Problem Solving

STEM projects or activities start with a problem, which can be real-life problem or a problem expecting to motivate students and brought to class by teacher. The very first step of STEM activities is to understand or determine the problem, which is the foundation of other steps. Lack of depth and comprehensive understanding of a problem situation could end with undesired results. Therefore, understanding the problem is essential to complete STEM projects. When this first step is ensured, changes on the rest would be acceptable with regard to situation, project, or activity, etc.

After understanding the problem, students would be expected to solve the problematic situation. Solving problem does not have to be related with mathematics because the definition of 'problem' is more generic. However, even though students do not solve any mathematical problem as content, they would still involve in problem solving process. Mathematical problem solving strategies were elaborated by Polya (1957) for mathematics: understanding problem, devising a plan, carrying out the plan, and looking back. These strategies "help an individual to understand a problem better or to make progress toward its solution" (Schoenfeld, 1985, p. 23) and when these strategies are examined, they are obviously vital steps to solve any kind of given problem. Thus, when students actually solve any problem situation, they actually use the ability of problem solving. Therefore problem solving is an essential, unignorable, and necessary ability to be used during STEM projects.

Mathematical Reasoning

In STEM practices, mathematical knowledge and understanding are inevitable elements. Reasoning -a component of these elements- was stated as "the principal instruments for developing mathematical understanding and for constructing new mathematical knowledge" (Ball & Bass, 2003, p. 30). The importance of reasoning was mentioned in mathematics and science standards from different aspects. For instance, reasoning and proof was one of the five process standards in Principles and Standards for School Mathematics; therefore, reasoning was emphasized several times in Common Core State Standards for Mathematics (CCSSM) as well. The idea of reasoning were also emphasized in Next Generation Science Standards Practices (NGSS) (e.g., constructing explanations and designing solutions, engaging in argument from evidence).

These standards (i.e., CCSSM and NGSS) not only shapes states' standards but also are used as resource while comparing other countries' standards. The stress on reasoning in both science and mathematics standards is an indicator of mathematical reasoning being a necessary element in STEM.

Mathematical reasoning was defined differently by researchers. Ball and Bass (2003) stated two types of reasoning: reasoning of inquiry and reasoning of justification. The former was a process when mathematical reasoning was used during exploration of new ideas and the latter was used during proving mathematical claims. Reasoning of justification in mathematics was consisted of two parts: the base of public knowledge and mathematical language (Ball & Bass, 2003). The base of public knowledge was basically defined as the knowledge known by and explicit for every individual in the community (e.g., students, teachers, mathematicians). Mathematical language was symbols, terms, representations etc. that was used to communicate in the community for clarity of mathematical ideas, claims and so on. Structural and process aspects of mathematical reasoning was elaborated in Jeanotte and Kieran's (2017) study. The structural aspects of mathematical reasoning were listed as deductive, inductive and abductive. The emphasis in these aspects were on being true, likely, or generating data and justification in the best way, respectively. Regarding process aspect of mathematical reasoning, searching for similarities and differences, validating, and exemplifying were other components (Jeanotte & Kieran, 2017). Generalizing, conjecturing, identifying a pattern, comparing, and classifying were listed as processes of mathematical reasoning related searching for similarities and differences. Validating, justifying, and proving dealt with the changing the epistemic value one way or another, modifying the epistemic value with data or support, and modifying the epistemic value with data or support from being likely to true, respectively (Jeanotte & Kieran, 2017). Lastly, exemplifying was defined as a mathematical reasoning process covers previous two process related aspects: searching for similarities and differences, and validation (Jeanotte & Kieran, 2017).

Generally as the last phase of STEM PBL activities ends with communication and reflection of students' outcomes (Capraro, Capraro, & Morgan, 2013), especially if engineering design process is followed. When students perform this last phase, they also use their reasoning ability. They need to explain their ideation and the reason why they choose to solve problem in their specific way. This process requires reasoning abilities such as justification, proving, and validation. For instance, when students were asked to explain their reasoning while solving given task, they used variety of representations such as analogy, diagrams, verbal or written statements to form conjectures, generalize, explain, validate and justify (Vale et al., 2017).

These types of representations were mostly used to perform engineering design process' steps during STEM PBL activities. Therefore, mathematical reasoning is an inevitable skill required in STEM education.

Mathematical Modelling

Mathematical modelling is almost an inevitable process in today's world. Mathematical modelling is "the process of solving problems set in the real world" (Berry, 2002, p. 214). It is actually a transition between real world and mathematics and during this transition the structure of real-life situations are probed through mathematics (Erbas et al., 2014). Researchers did not have a consensus about perspectives on this topic but they agreed on that mathematical modelling is in need when real word situation problems are solved (e.g., Berry, 2002; Blum & Borromeo Ferri, 2009; Niss, 2012).

Mathematical modelling was classified according to its usage in problem situations- as a vehicle or content. When mathematics was used as an aid to introduce or understand other curricular materials, it was used as vehicle; when mathematics was used to learn, improve and apply mathematical knowledge to solve real life problems, it would be used as content (Galbraith, 2012). In addition to that Kaiser (2005) and Kaiser and Sriraman (2006) classified different perspectives of modelling and listed as: a) realistic or applied, b) contextual, c) educational, d) socio-critical, e) epistemological and f) cognitive. Even though different explanations existed in the literature, the modelling process was defined almost similar by researchers.

Mathematical modelling process definition had variaty (Galbraith, 2012; Galbraith & Stillman, 2006; NCTM, 1989; Pedley, 2005) in literature however fundamental phases were similar and as following: 1) understanding the given situation; 2) formulating a mathematical model; 3) analyzing and solving the model; 4) comparing model with reality and validating; 5) revisiting the process if necessary. When STEM activities' design process was examined, STEM design phases included most of them (Capraro et al., 2013), so these steps are subpart of STEM design process. Therefore, even though STEM projects does not include mathematics as content in it, mathematical modelling would be part of these projects, thus 'M' would be automatically included.

Conclusion

STEM education is a popular subject for many countries, however we all know applying STEM education truly in schools require many changes. These changes extend from curriculum to teachers. When we think about all these requirements to establish well-applied STEM activities, hierarchically curriculum could be listed at the top. However it is so remarkable not to come across with connections among standards of STEM subjects in curriculum, whereas for instance most science standards already include mathematics and mathematical procedure in it. Therefore even though assuming not to include mathematics as a subject in STEM activities seems possible, when science or other two disciplines involves in STEM projects, mathematics indirectly involves as well. Thus, the importance of 'M' in STEM is impossible to ignore.

In this chapter, how "Mathematics" in STEM would be an unavoidable part of STEM and how it should be perceived other than being a subject was emphasized, because mathematics is "the world's single largest educational subject ... applied in a multitude of different ways in a huge variety of extra-mathematical subjects, fields and practice areas" (Niss, 2012, p. 49). Studies revealed that mathematics is a very important subject for students' STEM degree or career choice. Students' mathematics achievement predicted their STEM degree attainment (Tai, Sadler, & Mintzes, 2006; Tyson, Lee, Borman, & Hanson, 2007) and mathematics was one of the important factors explaining STEM career choices (Nicholls, Wolfe, Besterfield-Sacre, & Shuman, 2010). In addition to that STEM environments in early years increased their mathematics achievement compared to others (Oner, 2015). These results showed that mathematics is an essential subject to shape students' future but this does not mean it is not possible to arrange a STEM activity not involving mathematics as subject (i.e. content) when it is needed. What I would like to emphasize is that notwithstanding 'M' in STEM is not obviously included in any activity, it is still would be used as 'mathematical thinking and understanding', 'mathematical problem solving', 'mathematical reasoning', and/or 'mathematical modelling'.

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