PROSPECTIVE ELEMENTARY MATHEMATICS TEACHERS' CONTEXTUAL, CONCEPTUAL, AND PROCEDURAL KNOWLEDGE: ANALYSIS OF SELECTED ITEMS FROM THE PISA

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ABSTRACT: The aim of this study is to investigate the difficulties, which Turkish prospective elementary mathematics teachers have in solving the Programme for International Student Assessment (PISA) 2012 released items. Data were collected from 52 teacher candidates through a 26 item-written test. The data indicated that PISA items could be categorized according to whether they require contextual, conceptual or procedural knowledge. Analysis of data also indicated that the participants encountered problems mostly in items requiring the combined contextual, conceptual, and procedural knowledge. Although many of the participants could produce correct answers for procedural knowledge items, several could not, and few were able to give mathematical explanations and make appropriate estimations for conceptual knowledge items. The results lead us to conclude that the prospective teachers' contextual knowledge was generally fragmented, and that the role of the type of mathematical knowledge identified by PISA items should be explored with particular attention to the function of contextual knowledge. Implications for teacher education related to the findings of the study are discussed.

Key words: Contextual knowledge, conceptual knowledge, procedural knowledge, PISA items, preservice teachers

INTRODUCTION

In recent years, much has been written about students' mathematics performance in international large-scale assessments such as Programme for International Student Assessment (PISA), Trends in International Mathematics and Science Study (TIMSS), and Progress in International Reading Literacy Study (PIRLS) (e.g., Andrews, Ryve, Hemmi, & Sayers, 2014; Roth, Ercikan, Simon, & Fola, 2015). There is ample evidence that the results of these cross-cultural comparative assessments caused a considerable stir in the worldwide educational community because of students' poor mathematics performance. Many articles suggested that mathematics scores were dropping out worldwide (e.g., Alphonso, 2013). This reaction is reflected in Turkey as well (Yücel, 2013), where mathematics education is either criticized or, if the country is placing bottom-down in the overall rankings, the distance to the achievement scores of the leading countries is noted as a major concern (Akyüz, 2014).

To this end, we focused on the *Programme for International Student Assessment (PISA)* initiated by the *Organisation for Economic Co-operation and Development (OECD)*. Scores on mathematical literacy items from the PISA is supposed to measure the extent to which students, at the end of their compulsory schooling (15 years), have acquired the mathematical knowledge and skills that are essential for everyday-life situations (OECD, 2006). The influence of such international assessments on both international (e.g., Boasen et al., 2014) and national education policy is considerable (e.g., Yıldırım, Yıldırım, Ceylan, & Yetişir, 2013). Results of these large-scale assessments are employed to develop curriculum planning and resource allocations. Henceforth, there is a need to analyze the items indepth and trace the difficulties encountered by the students in order to make educational decisions.

Simply put, the result was major debates pointing out the direct attention to the practice of teaching: if there is a need for effective learning then effective teaching is necessary. In this regard, researchers put their efforts in investigating what constitutes the mathematical knowledge for teaching and whether teachers represent that knowledge in a productive way (Hill et al., 2008). Drawing on these assertions, we assume that for teaching any mathematical knowledge the first requirement is to possess the knowledge. We suggest that this knowledge is not independent of tasks, and therefore the type of mathematical knowledge needed to reach a solution for a particular task. Accordingly, we carried out a study aimed at discovering the mathematical knowledge of students training to be elementary mathematics teachers when confronted with PISA items. Drawing on the results from PISA, researchers have shown that mathematical performance may operate differently across cultures (Chiu & Xihua, 2008; Chiu & Klassen, 2010; Kriegbaum, Jansen, & Spinath, 2015), but few studies have compared the preservice teacher performance in the particular released items in their analyses or explored categorizations of these items

concerning the type of knowledge they require (e.g., Olande, 2014; Sáenz, 2009). Examining the difficulties in PISA items might yield more indepth understanding of prospective teachers' mathematical knowledge that can be productively applied to problem situations.

The purpose of our study was to investigate the difficulties that preservice elementary teachers encountered while performing on PISA tasks that require using different types of mathematical knowledge.

Theoretical Background

Recent research studies have shown that structured, organized knowledge enables individuals to perform tasks successfully and remember more relevant information than if we have only memorized isolated mathematical facts and/or automatized procedures (Bransford, Brown, &, 2001). According to Rittle-Johnson and Koedinger (2005), organized knowledge requires individuals to integrate their contextual, conceptual, and procedural knowledge within a content domain. In our study, we propose that information on these three types of knowledge will provide the whole spectrum of the context of PISA tasks, and provide empirical verification of the knowledge required for solving these tasks.

Types of Mathematical Knowledge

In literature on mathematics learning and instruction knowledge plays a crucial role and is attributed a wide variety of types and properties. Types of knowledge are important for problem solving (Rittle-Johnson & Alibali, 2005), and that components of the knowledge base are characterized by the *function* they fulfill in the performance of a specific task (Gott, 1989). Mathematical knowledge can be thought of as consisting of three complementary types: contextual, conceptual, and procedural. A number of researchers offered definitions for both conceptual and procedural knowledge (e.g., Hiebert & Lefevre, 1986) and/or for the entire contextual, conceptual, and procedural knowledge (e.g., Rittle-Johnson & Koedinger, 2005).

Contextual knowledge is knowledge of how things work in daily life situations in the real world, which develops from our informal interactions with the physical world outside (Leinhardt, 1988; Saxe, 1988). Students' contextual knowledge can be elicited by presenting problems in story contexts (i.e., real-life settings), which are very likely to be unfamiliar. A large body research shows that many students have substantial difficulties in solving such non-routine problems (Lesh & Zawojewski, 2007; Schoenfeld, 1985; Verschaffel, Greer, & DeCorte, 2000). In relation to the present study, it is of value to take into consideration that the PISA strongly emphasizes the need to develop students' capacity to use mathematics in context, and thus stipulates contextualized tasks, which involve personal, occupational, societal, or scientific situations encountered in the daily life of a modern society. Contextual knowledge tasks in PISA, therefore require students to 1) translate from a real-world setting to the domain of mathematical structure and representations (i.e., regularities, relationships, and patterns), 4) identify constraints and assumptions by mathematical modeling, 5) use appropriate variables, symbols, and diagrams for representing an everyday situation mathematically, 6) explain the relationships between the context-specific language of a problem and the formal language of mathematics, and 7) translate a problem into mathematical language or representation.

Conceptual knowledge is knowledge of concepts or principles and involves rich connections (de Jong & Ferguson-Hessler, 1996). It is characterized most clearly as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information (Hiebert & Lefevre, 1986, p. 3). It is widely acknowledged that conceptual knowledge is essential for problem-solving (e.g., Rittle-Johnson & Koedinger, 2005). Conceptual knowledge tasks in PISA, therefore require students to 1) understand the extent and limits of mathematical definitions, facts and principles, 2) understand the extents and limits of mathematical solutions, 3) critique and identify the limits of the model used to solve a problem, 4) explain whether a mathematical conclusion makes sense in the real world context, 5) evaluate the reasonableness of a mathematical procedure, and 7) make contextual judgments about how the mathematical results should be adjusted or applied.

Procedural knowledge is the knowledge of subcomponents of a correct procedure (Rittle-Johnson & Koedinger, 2005). It is defined as knowledge that is memorized by rote with some computational skills (Baroody, 2003). From this description, the essence of procedural knowledge is that it involves applying sequential action steps and automatized techniques for solving problems (Bisanz & Lefevre, 1990). Procedural knowledge tasks in PISA, therefore require students to 1) devise and implement strategies for finding mathematical solutions, 2) apply mathematical algorithms and structures when finding solutions, 3) manipulate numbers, algebraic expressions and

equations, and geometric representations, 4) manipulate graphical and statistical data and information, 5) construct diagrams, tables, and graphs and extract relevant mathematical information, 6) use and switch between different representations in the process of finding solutions, and 7) make generalizations based on the results of applying mathematical procedures.

Obviously, mathematical knowledge is complex in nature. Although the abovementioned definitions seem to be clear, mathematical knowledge may not be easily distinguished as one kind or to the other (Hiebert & Lefevre, 1986). Henceforth, meaningful knowledge of contexts, concepts, and procedures can be viewed as intricately and necessarily interrelated, not as distinct categories of mathematics (Star, 2005).

METHODS

Participants and Procedure

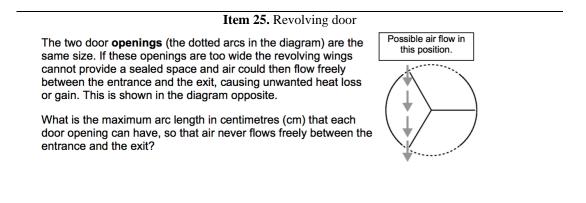
The sample consisted of 52 preservice teachers (38 females and 14 males) from Mersin University, Turkey. The participants were third-year (junior) students attending to Faculty/School of Education who were majoring in Elementary Mathematics Education. They had taken several courses in mathematics (e.g., Calculus, Linear Algebra) and mathematics education (e.g., Methods of Teaching Mathematics). Participants had no prior experience of the international assessment tests such as PISA.

The data were collected by the second researcher in the Spring semester of 2015-2016 academic year. A 26-item mathematics test which was originally released by the OECD (2013) was distributed to the participants as a booklet. The items adapted into Turkish, were released by the Ministry of National Education [MoNE], General Directorate of Assessment Evaluation and Testing Services (GDAETS, 2012, available from http://pisa.meb.gov.tr/?page_id=617). Participation was voluntary. The testing time for the entire mathematics test was approximately 90 min. Participants were also informed that the test was not to be considered as assessment and that; it would not be seen by other university staff.

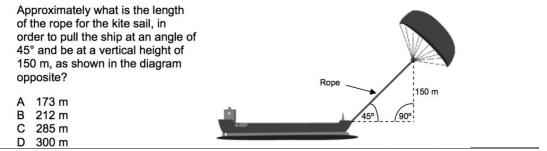
Data Collection and Analysis

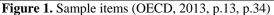
In this study, problems were categorized using the definition of contextual, conceptual, and procedural knowledge developed by Rittle-Johnson and Koedinger (2005). Each problem was coded so that it was included into one or more of the following three categories. The authors and another mathematics educator conducted the coding. Before each rater individually coded the problems, they discussed the interpretation of the categorization, which is based on the definitions of contextual, conceptual, and procedural knowledge. They met regularly to compare codes and identified the rationales for the categories. Although there was a significant overlap in the rationales, they discussed the different opinions about the problems reflecting types of knowledge. Afterwards, they separately coded the items and, met and discussed each of the coding discrepancies coming to 100% agreement for the data.

To clarify and discuss how we determined the type of knowledge, we present examples of items according to three type of knowledge: contextual, conceptual, and procedural (see Figure 1).



Item 8. Sailing Ships





In Figure 1, Item 25 (*Revolving Door*) demands contextual, conceptual, and procedural knowledge because item provides everyday problems in the real world and non-routine applications (contextual knowledge), includes relationships and connections (conceptual knowledge), and requires calculation the length of an arc (procedural knowledge). Moreover, Item 8 (*Sailing Ships*) demands conceptual and procedural knowledge considering that it requires interpreting the visual representation of diagram (conceptual knowledge) but not the context of the problem and using Pythagorean Theorem (procedural knowledge). Test carried out by the prospective teachers and the difficulty of items was classified according to the percentage of correct answers to each one. In order to identify the errors according to the deficiencies in knowledge type, the responses of prospective teachers were analyzed further.

RESULTS AND FINDINGS

Table 1 presents a classification from more to less difficulty of the tasks. Column 3 indicates the type of knowledge determined by the group of experts: contextual (Ct), conceptual (C), and procedural (P). Column 4 shows the percentage of correct answers given by the prospective teachers.

Item No	Task	Type of Knowledge	Percent Correct
25	Revolving door	Ct, C, P	5.8
23	Garage	Ct, C, P	13.5
9	Sailing ships	Ct, C, P	23.1
18	Helen the Cyclist	Ct, C, P	38.5
3	Drip rate	Ct, P	46.2
8	Sailing ships	C, P	46.2
14	Climbing mount Fuji	Ct, C, P	46.2
15	Climbing mount Fuji	C, P	48.1
1	Apartment purchase	Ct, C	73.1
26	Revolving door	C, P	73.1
17	Helen the Cyclist	C, P	75
20	Which car?	Р	80.8
22	Garage	Ct, C	82.5
11	Ferris Wheel	Р	82.7
12	Ferris Wheel	C, P	82.7
13	Climbing mount Fuji	C, P	82.7
21	Which car?	Р	82.7
24	Revolving door	Р	82.7
10	Sauce	Р	84.6
6	Charts	C, P	86.5
16	Helen the Cyclist	C, P	86.5
7	Sailing ships	Ct, C, P	88.5
5	Charts	Р	90.4
2	Drip rate	Ct, C	92.3
4	Charts	Р	92.3
19	Which car?	Ct, C	94.2

Table 1. Classification of the tasks by knowledge type and level of difficulty

As seen in Table 1, an examination of the percentage of the prospective teachers who answered the tasks indicated that they encountered problems mostly in tasks requiring the combined contextual, conceptual, and procedural knowledge. Although many of the prospective teachers tended to outperform the tasks requiring procedural

knowledge, as expected, several could not, and few were able to make appropriate estimations for tasks requiring conceptual knowledge. For example, the most common incorrect answers given by the prospective teachers were included in task 25, 23, 9, 18, and 3. This was a valuable piece of information for the detailed explanation for the follow-up analysis of prospective teachers' difficulties related to the tasks in relation to the type of knowledge. Thus, we analyzed the most common errors and the difficulties with these tasks in the following sections.

Task 25: Deficiencies in Contextual Knowledge

Analyses of the errors in the most difficult tasks revealed that most of the prospective teachers had difficulty in understanding the context of the problem. In particular, most of the difficulties arose in the case of interpreting of geometrical model of a real-life situation, which was unfamiliar in the school mathematics. Task 25 posed an original problem whose solution demands non-routine applications rather than specific conceptual and procedural knowledge. The answers were in the range from 103 to 105 and 1/6th of the circumference was also accepted as another response. The most common incorrect response categories were $200\pi/3$. This error occurred while they misinterpreted the door openings (i.e., the dotted arcs in the diagram) and followed an incorrect procedure by calculating the length of an arc. In brief, prospective teachers showed a lack of interpreting and handling non-routine applications (or non-standard problem).

Task 23 and Task 18: Deficiencies in Conceptual Knowledge

There were some deficiencies in basic conceptual knowledge and some difficulties in elaborating visual aspect of the model in the problem. Task 23 poses an original problem whose solution demanded interpretation of a house plan and calculation of the total area of the roof using the Pythagorean Theorem. The complete responses were any value from 31 to 33. Moreover, the correct use of Pythagoras Theorem with calculation error and incorrect length used $\frac{1}{500}$ was also accepted as a partial response. The most common incorrect response categories were $17m^2$ and $2.5m^2$. These errors occurred while they were not able to elaborate the 3D view of the roof. As shown in the Figure 2, one of the prospective teachers drew the area of the roof and calculated the area of a triangle for the total area of the roof. This might explain the difficulty of elaborating the visual representation of the roof.

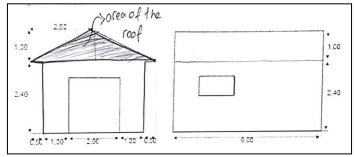


Figure 2. Example from prospective teachers' solution

Another important finding for the difficulties with the conceptual knowledge showed that the main error made by the prospective teachers consisted in calculating an average speed over two trips given two distances travelled and the times taken in task 18. For this task, the most common incorrect response categories were 20 and 85/3. They made an error in calculation of average speed considering that they calculated the average speed as an arithmetic mean of two speeds (i.e., they wrote the formula, (V1+V2)/2). This might be an indication of the deficiencies in basic conceptual knowledge.

Task 3: Deficiencies in Procedural Knowledge

Analyses of the errors showed that common difficulties and errors appeared in handling of formulae and calculating of equations, which revealed insufficient procedural knowledge. Task 3 required the handling of an equation (D=dv/60n) and substituting two given values. The answer of 360 could be obtained by a simple calculation of the volume in mL of the infusion. The most common incorrect responses were 6 and 90. These errors arose while many prospective teachers were not able to transpose the equation. In brief, they had difficulty in handling the variables and calculating the equations.

Task 9: Deficiencies in Integrated Knowledge

From the point of view of errors, there were three kinds of knowledge (i.e., conceptual, contextual, and procedural knowledge), which were of basic importance for Task 9. This task was based upon solving a real work situation involving cost saving and fuel consumption. It demands at formulating change and relationships. The answers were ranged from 8 to 9 years with adequate mathematical calculations. The most common incorrect response categories were 2 years, 3.5 years, and 4 years. Prospective teachers could not suggest a general relation between diesel consumption per year and cost for diesel without a sail. This shows a lack of understanding of conceptualization of connections among the cost savings and fuel consumption. Moreover, the problem containing very detailed information about sailing ship (e.g., the length, breadth, load capacity, and maximum speed of the sailing ship) and including the term "zed" was perceived as a negative aspect to the context. It was evident that the difficulties might arise in the case of sailing ships, which were unfamiliar to the participants.

CONCLUSION

Beyond the well-researched phenomenon of conceptual and procedural knowledge, contextual knowledge has received relatively little attention within educational research. Findings of the present research showed that the majority of preservice teachers have rudimentary contextual knowledge and that they are not proficient in utilizing that type of knowledge while solving non-routine tasks. In a related vein, we also showed that assessment of the mathematical knowledge of inservice teachers includes assessment of the extent to which they have contextual, conceptual, and procedural knowledge, which they can productively apply to real-life problems in novel situations. Our data also support the assumption that a preservice teacher's mathematical knowledge is not only built on knowledge of concepts (i.e., conceptual knowledge) and knowledge of procedures (i.e., procedural knowledge), but also on the composition of the knowledge types in terms of knowledge of concepts and procedures, and on the knowledge of contexts (i.e., contextual knowledge). This information is of particular value for teacher educators, for whom the subject matter knowledge of their students should be a desired educational goal in itself.

RECOMMENDATIONS

The simple message from this research is that preservice teachers encounter difficulties with performing tasks that require using integrated mathematical knowledge that is the blend of contextual, conceptual, and procedural knowledge. Within this amalgam, contextual knowledge is particularly important because it involves building relationships between daily life experiences and preexisting mathematical knowledge, and further transferring knowledge in a novel situation (Crowford, 2001). The utopian view would be for all preservice teachers to harmonize contextual, conceptual, and procedural knowledge; however, this is unlikely as it is almost inevitable that most preservice teachers tend to activate procedural knowledge while solving mathematical tasks. A more pragmatic solution might be for teacher education programmes to include more emphasis on the acquisition of contextual, conceptual, and procedural knowledge, and how these knowledge types are tied up with preservice teachers' understanding of particular mathematical content. On the side of the continuing professional development of inservice teachers a challenge might be that how appropriately acquired contextual, conceptual, and procedural knowledge is used by the teachers to develop students' mathematical thinking. The hope would be for preservice and inservice teachers to be able to grasp various aspects of acquiring contextual, conceptual, and procedural knowledge, understand the mathematical knowledge capacity of individual students, and guide them about how to mobilize knowledge types in the most productive ways. It is important that preservice teachers should understand the potential of their mathematical knowledge, types of mathematical knowledge in particular, to influence their future profession in the classrooms as inservice teachers.

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