Classroom Assessment of Mathematical Modeling Tasks

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Introduction

Mathematical modeling and modeling in general has grown in popularity in school mathematics and other STEM areas (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State Officers [CCSSO], 2010; Consortium for Mathematics and Its Applications [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2016). For instance, modeling has been included in assessments and documents such as the programme for international student assessment (PISA), national assessment of educational progress (NAEP), and guidelines for assessment in mathematical modeling education [GAIMME] report. Standards of education in mathematics and science place an increasing emphasis on modeling, where students are engaged in analytical thinking , reasoning, critical thinking, and problem solving skills (see National Council of Teachers of Mathematics [NCTM], 2014; Next Generation Science Standards [NGSS Lead State, 2013]; NGA Center & CCSSO, 2010). The growing awareness of the importance of modeling as an integral part of mathematical competencies for students requires thoughtful assessment of classroom mathematical modeling tasks.

The role of assessment in mathematical modeling is critical and complex. However, the use of modeling tasks in most classrooms is limited (Blum, 2015). One reason is that mathematical modeling is a challenging task that requires several competencies and skills including problem posing and problem solving. Additionally, mathematical modeling requires real-world knowledge from certain domains that may not be familiar to most teachers of mathematics in the classroom, and makes solutions less predictable and linear (Leong, 2012). Moreover, anecdotal experiences show that a common and frequent challenge that most teachers of mathematics face is the assessment of mathematical modeling tasks. According to the GAIMME report, mathematical modeling tasks do not always result in a simple, precise answer or the right solution and the literature on classroom assessment of modeling is scarce. Therefore, the question among most teachers of mathematics and even mathematics educators is how do we assess classroom mathematical modeling, and in particular, the modeling process?

To address this question, this article offers useful information with two purposes in mind. First, we emphasize the importance of formative and summative classroom assessments of mathematical modeling.

Second, we introduce an assessment framework developed by the authors with the potential to assist in assessing mathematical modeling tasks. We view assessment as an integral facet of teaching. According to the National Research Council (NRC, 2001), these are the three main purposes of assessment in student learning: a) to assist student learning (formative assessment), b) to measure individual achievement (summative assessment), and program evaluation. For the purpose of this article, we focus our attention on formative and summative assessments. Formative assessments support learning, whereas summative assessments measure achievement. Formative classroom assessments of mathematical modeling should be based on tasks that bring into focus modeling techniques, modeling strategies, and the steps in the modeling process that provide scaffolding for student learning of these aspects of modeling.

Formative assessment provides "information to be used as feedback to modify teaching and learning activities" (Black & Wiliam 1998, p. 104). Research related to teachers using formative assessments indicated changes in their assessment practices, which resulted in increases in student achievement (Black, Harrison, Lee, Marshall, & Wiliam, 2004). In this way, teachers and students can identify the level of students' progress and where development is still needed. An important type of summative classroom assessments are open-ended tasks that require modeling techniques, modeling strategies, and all of the steps in the modeling process; these can be graded using a holistic approach guided by a rubric. Therefore, this article provides a framework, the rationale for, and examples of these formative and summative assessment of mathematical modeling task.

A Brief History and Purpose of Mathematical Modeling

Emphasis on mathematical modeling in school mathematics has increased in the half century since Pollak noted that "many teachers of mathematics have never been involved in the process of building mathematical models of situations in the outside world" (Pollak, 1966, p. 122). Since 1980, COMAP has produced materials to help teachers teach mathematical modeling. For the past decade, PISA has encouraged the implementation of mathematical modeling worldwide (OECD, 2003), and the Moody Foundation and SIAM have promoted mathematical modeling through the Moody Mega Math (M3) challenge. Since 2010, the Common Core standards (NGA Center & CCSSO, 2010), have called for students to "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (p. 7). Today there are instructional materials that incorporate mathematical modeling (e.g., COMAP, 2012, 2013); large-scale assessment of mathematical modeling (e.g., PISA, PARCC), and contests to assess mathematical modeling prowess (e.g., the M3 Challenge). More recently, COMAP and SIAM collaborated to develop guidelines for assessment and instruction in mathematical modeling education [GAIMME] report, (COMAP & SIAM, 2016), yet there is limited guidance to assist teachers of mathematics in assessing mathematical modeling.

To help bridge this gap, we focus on ways to help teachers to apply formative and summative assessment techniques to support learning and to measure performance on mathematical modeling tasks. According to the authors of the GAIMME report, mathematical modeling is “a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena” (COMMAP & SAIM, 2016, p. 8). For us, mathematical modeling is a process that uses mathematics to represent, investigate, and reach conclusions about situations in the world around us. Such activities not only motivate students but also enrich their understanding of the underlying mathematics. The purposes of mathematical modeling include (a) supporting student interest in the learning of mathematics (Blum & Borromeo Ferri, 2009; Pollak, 2003), (b) making mathematics relevant and meaningful to students (English & Watters, 2004; Flavres & Schiff, 2013), (c) providing students the opportunities to improve their problem-solving skills and mathematical abilities (English, 2007; Lesh, 2012), and (d) improving student achievement in mathematics (Boaler, 2001; English, 2007).

Mathematical modeling as a process involves mathematization, interpretation, and communication. According to Pollak (2011), mathematical modeling is a process that teaches students how to use mathematics in everyday life and in intelligent citizenship. A crucial aspect of teaching mathematical modeling is to help students internalize the mathematical modeling process. Extending beyond traditional word problems, mathematical modeling focuses on the iterative nature of the modeling process and provides multiple point of entry (Blum, 2011). Modeling requires knowledge of contexts beyond the classroom that may not be familiar to students and teachers, and modeling problems have solution paths that are less straightforward and predictable than traditional word problems (Pollak, 2011; Zbiek & Conner, 2006).

The Assessment Triangle and Assessing Mathematical Modeling

In our approach to classroom assessments of mathematical modeling tasks, we employ the elements of the assessment triangle, which has three interactively linked elements: a cognitive model of the learning goal, a lens for observation (often a problem or task), and a scheme for interpreting the observed student work (NRC, 2001). Figure 1 illustrates the interconnectedness of these elements: cognition, observation, and interpretation. In a typical classroom, the teacher observes the work of students and interprets their understanding of the content being taught based on the amount and quality of that work. Often the underlying cognitive model is implicit or absent.

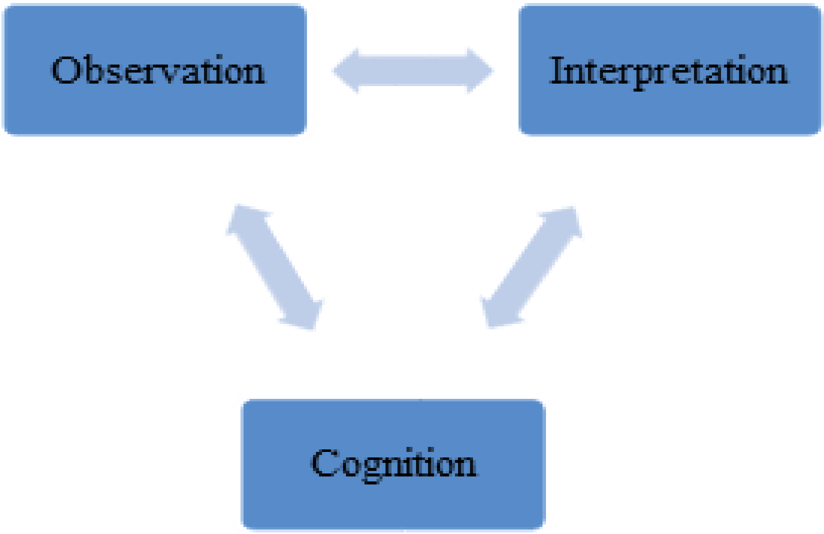

Figure 1. Assessment Triangle (adapted from the NRC 2001, p. 44)

In the assessment triangle model, cognition refers to the theory and set of beliefs about how students learn and represent knowledge to demonstrate competence. Below we present a schematic representation of mathematical modeling process to serve this purpose. Observation requires a description for assessments tasks that will elicit illuminating responses from students—the modeling tasks that allow the teacher to observe student learning (NRC, 2001). When assessing students modeling performance, the teacher monitors what the students say, do, and write. Interpretation encompasses all of the methods and tools used in reasoning about the task including assumptions, choices, and models developed. The interpretation shows the reasoning one makes from the student's evidence of learning. The assessment triangle model indicates that having all the three elements (cognition, observation, and interpretation) working together can provide an effective assessment framework. An explanation is that in the case of solving a modeling task, cognition will be clearly identifying the construct being learned and explicitly conceptualizing a cognitive model for the task. Next, and through observation, the modeler has to identify tools used to elicit the knowledge and skills associated with the construct being assessed. Then, the modeler has to make a decision based on the evidence from the observation tools through interpretation. All three are intertwined and provide a strong framework for assessing modeling tasks. According to Lyon (2011), the alignment between the elements can help identify whether teachers are assessing topics called for in STEM education reform efforts and if their tasks can actually provide evidence of student learning.

Assessment and Mathematical Modeling Performance

To have a clear knowledge as to how to assess mathematical modeling performance requires an understanding of the modeling process. In this section, we present a modeling process adopted from Blum and Leiss (2007) as a cognitive framework in assessing mathematical modeling performance. Because of the iterative nature of the modeling process, assessment of intermediate models and modeling process is crucial to progress toward an increasingly effective and efficient model (Zawojeski, 2016). The mathematical models students generate and the mathematics they bring to bear during the modeling process, provide a window into their mathematical ways of thinking (Diefes-Dux, Hjalmarson, & Zawojewski, 2013). According to Pollak (2003), the modeling process is where one identifies a situation in the real world, makes certain assumptions, and then uses a mathematical model to obtain a mathematical formulation to get results that can be translated back into the real world to validate the practicality of the results. Thus, the modeling process is a translation between the real world and mathematics in both directions.

The mathematical modeling process is a building link between mathematics as a way of making sense of our physical or social world and mathematics as a set of formal representations. The modeling process helps students comprehend mathematical concepts and teach them to formulate and solve specific situation-problems. Because mathematical modeling is an iterative process that involves elements of both a treated-as-real world and a mathematics world, several researchers have cited and used Blum and Leiss (2007) seven stages modeling process, while others have mentioned and used the CCSSM (NGA Center & CCSSO, 2010) six phase modeling cycle. There are three main reasons why the modeling process needs to be part of the teaching and assessing of mathematical modeling tasks: Modeling process a) offers students an understanding of what mathematical modeling means b) gives students orientation within their modeling process, and c) allows students to think about their modeling process and on a metacognitive level (Borromeo Ferri, 2018).

The slight variation in the modeling process is dependent on various directions and approaches of how mathematical modeling is understood and the task(s) employed. Zbiek and Conner (2006) explained that mathematical modeling is an iterative process involving revisions before one arrives at an acceptable conclusion and it involves movement among elements such as the real world situation, a mathematical entity, and a mathematical solution. We present a seven step modeling process or framework based on ideas from the two aforementioned modeling processes and Hall's (1984) modeling process. We did not explicitly use the modeling cycle described in the Common Core because the distinction between reality (the rest of the world) and mathematics was unclear as specified in other modeling processes. The unique feature of the modeling process presented in this article takes into account reality, the heuristic for teaching modeling, and the assessment framework for measuring mathematical modeling performance.

The modeling process depicted in Figure 2 describes and provides the necessary information used as the framework in assessing modeling performance. The cyclical nature of Figure 2 highlights the iterative nature of the modeling process. The phases in this modeling process include (a) understand the task, (b) setup a model, (c) devise the mathematical problem (mathematizing), (d) solve the mathematical problem, (e) interpret solution, (f) validate the model, and (g) report conclusions. Drawing on the modeling process, assessment of students modeling performance suggests two approaches: (a) holistic approach and (b) the different phases of the modeling process. Both methods of assessing modeling performance describe formative and summative forms of assessment. Additionally, targeting the phases in the modeling process warrants giving students feedback at all times whiles working on a modeling task, and this characterizes formative assessment of modeling performance.

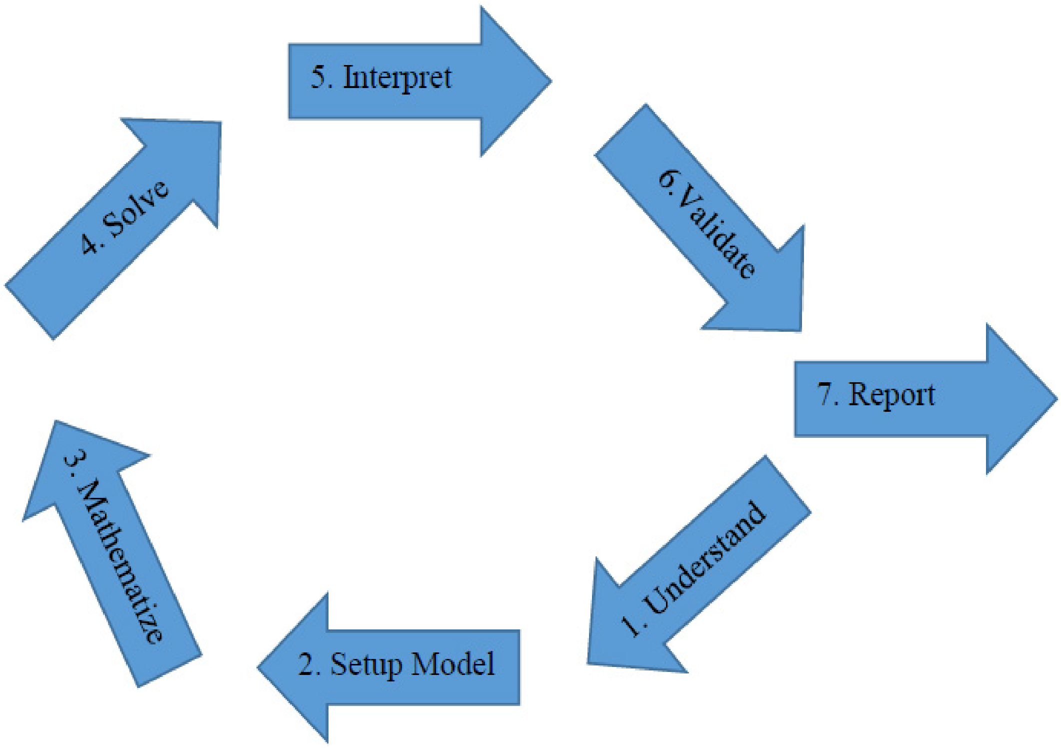

Figure 2. A Schematic Representation of Mathematical Modeling Process

Contextualizing the Modeling Process: The Hoodie Problem

To have an understanding of how the modeling process works, we discuss a specific modeling task in a context that most students will relate to in the hoodie problem as shown in Figure 3. A possible solution path is offered along with explanations that highlight the modeling process.

Hoodie Problem

You visit Egglawn University, and you wish to buy a hooded sweatshirt (hoodie) for your campus tour. An Egglawn hoodie costs $30 in shops uptown, near campus, and $35.00 downtown, near your residence. Is it worth going uptown to buy your hoodie?

Figure 3. Contextualizing Modeling: The Hoodie Problem.

Understand the Task. This phase requires the student to have some understanding about the task. Some knowledge and familiarity with cost, distance, and time functions will help in developing a model. At this phase in the modeling process, the following questions will help the problem solver: Do I know enough about the real situation? How do I model the situation? What is the purpose of the model? Additionally, the context of the task will generate some discussions to the solution process.

Setup a Model. At this phase in the process, the student need to consider which features of reality are to be included–making assumptions and choices—and choose a model that will serve the purpose in solving the problem. In this particular task, a good assumption will be the type of vehicle the student will be driving to go uptown. Other features for the student to consider might include the vehicle consumption, gas price, and distance between the two locations. Students can be guided at the meta-level to think about what information they need to determine which location will be a better buy and whether they have been provided with all the information.

Mathematize. Mathematizing the task at this phase in the modeling process is about the student deciding what to use–equations, graphs, symbols, technology, variables, diagrams, etc.–to move the problem into a mathematical world. Based on the assumptions mentioned in phase 2, one could create or come up with the following mathematical representations:

C: Downtown = $35.00, C: cost of hoodie.

C: Uptown = $30.00 + 2×d×c×g, d: distance, c: consumption, and g: gas price

C: Uptown < C: Downtown?

Solve the Mathematical Problem. At this phase, the student works mathematically to solve the mathematical model and problem based on the assumptions. One way to find a solution is to find the cost of buying the hoodie uptown. C: Uptown ≈ $30.00 + $2.17 = $32.17. The mathematical results shows that C: Uptown < C: Downtown!

Interpret Solution. This phase is about interpreting outcomes. On the basis of the results, the student has to interpret the mathematical results in the real world situation. The student must check to ensure that the problem has been answered within the assumptions that have been made. Interpretations made require explicit assumptions and initial conditions. This will help students to understand that solutions to problems are limited by the context and are not easily transferable to other situations. Furthermore, does the mathematical model behave reasonably when conditions are changed? The student will discuss the results and will need to decide whether it is cheaper to buy the hoodie uptown or downtown.

Validate the Model. At this phase, the student has to discuss the strengths and weaknesses of the model and how it relates to the real world problem. The statement that "all models are wrong, but some are useful" is a reminder about oversimplification and ignoring underlying assumptions. The question that can be asked here could be: Is it worth it to drive about five miles to save $2.83? Have we considered time, pollution, and risk involved to drive five miles? At this phase three things: evaluation, validation, and if possible iteration have to be considered. Evaluation addresses the situation whether the model fulfils its purpose. Validation focuses on comparing the conclusions with reality. Iteration comes into play when the mathematical model does not behave reasonably in comparison to the real world problem. In such a situation then the modeler (student) has to go back and check on the assumptions (phases 1 & 2). However, if the conclusions are acceptable, then the student moves to the final phase by reporting conclusions.

Report. This is the final phase and a valuable part in the modeling process where students use language and communication skills to express their mathematical ideas. It is at this phase that we can elicit and observe evidence of students' mathematical thinking. Reporting conclusions and the rationale behind them both in an oral and written form will help students develop communication skills needed for the 21st century. The report may include documentations of the students' progress through the phases of the modeling process and their final answers. Figure 4 illustrates a typical written report based on the hoodie problem.

One could go from downtown to uptown by car; distance (one way) is about 5 mi, consumption 1 gal/18 mile, and gas cost $2.45/gal.

C: Downtown = $35.00

C: Uptown = $30.00 + 2d×c×g

C: Uptown ≈ $30.00 + $2.17 = $32.17

C: Uptown < C: Downtown!

Result: It is cheaper to buy the hoodie uptown!

However, drive 10 miles to save $2.83?? Time?? Risk, Air pollution??

No, that is not worth driving, so we buy the hoodie downtown.

Figure 4. A Sample Solution Path to the Hoodie Problem

Formative use of Summative Assessment in a Modeling Task

Much work has been done in developing ways in which summative assessment of mathematical modeling task are carried out in various ways (Wake, 2009). Although this has had little impact on the general assessment of modeling tasks, the framework and structures developed in this article may well provide structures to inform formative assessment of modeling tasks in classrooms. The main goal of designing formative assessment is to assist learning. Consequently, formative assessment of modeling often focus on individual steps or strategies within the modeling process. In formative assessment, scaffolding can be provided within different phases of the modeling process, which allows students to learn about and compare models to get used to making several iterations around the modeling cycle and comparing the results as a means of validation. The Children in Hot Cars problem (adapted from Galbraith & Holton, 2018) below is an example to illustrate formative assessment of a modeling task.

The mix of formative and summative assessment of modeling tasks help students to show progress in problem solving and critical thinking skills. Formatively assessing students on this task through questions such as "what did you do here?, why did you do that?, or explain why you chose this approach? and scaffolding help elicit student thinking on the task. The information gathered through formative assessment may guide selection of future tasks and provide collective data that can be complied to yield a summative grade for students. This Children in Hot Cars problem will be used as a context to illustrate how students as modelers engage in formative use of summative assessment in modeling tasks:

> *Suppose you are asked to investigate why small children are so much at risk in locked cars in hot weather conditions, how would you solve this problem*?

Because modeling provides an opportunity for students to document and evaluate a wide range of conceptual understandings that may otherwise be difficult to document, we argue that the assessment of modeling tasks need to be embedded in the modeling process, rather than considered as an independent process (Eames, Brady, & Lesh, 2016). Formative assessment occurs during teachers' observation and interpretation of student models, which has the potential of improving instruction. Additionally, teachers engage in summative assessment when they evaluate the final models for effectiveness and efficiency to meet the criteria and constraints of the real-world modeling task. The assessment of the task associated with the modeling process may include a) formulating situations mathematically, b) employing mathematical concepts, facts, procedures, and reasoning, and c) interpreting and evaluating mathematical outcomes. (Zawojeski, 2016). Below is a typical response to the task with a mix of formative and summative assessment techniques embedded in the modeling process.

Understand the Task: In this phase, the modeler describes the real-world problem with the goal of specifying the mathematical problem.

Setup a Model: Here, the modeler tries to formulate the mathematical model. The modeler must consider which features of reality are to be included–making assumptions and choices—and choose a model that will serve the purpose in solving the problem.

Mathematize: For this specific task, one can assume the rate of fluid loss from a body depends on (is proportional to) its surface area, SA. Also, the amount of fluid in a body depends on (is proportional to) its volume, V. So a critical factor to consider, is the surface area/volume ratio of the body. Suppose we use a unit cube (see Figure 5) to represent the body of a person:

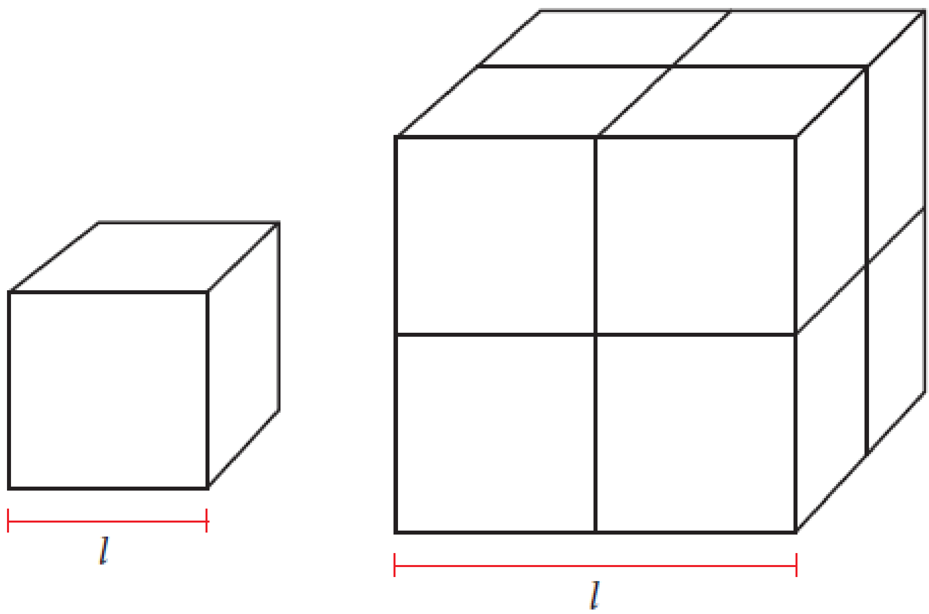

Figure 5. Sample Small and Large Cubes (adapted from Galbraith & Holton, 2018).

Then, SA of small cube = 6; V of small cube = 1; SA/V small cube = 6

SA of large cube = 24; V of large cube = 8; SA/V large cube = 3

Solve the Mathematics: Creating a table (see Table 1) of the ratio of blocks with lengths from 1 to 10, shows a pattern that smaller cubes have higher surface area/volume ratios than larger cubes. This indicates that smaller cubes have a greater surface through which to lose fluid relative to volume of fluid they have to lose. Thus, smaller cubes will lose fluid at a greater rate than larger cubes.

Table 1. A Table Showing Ratios of Block with Different Length Size.

Side Length of Cube	1	2	3	4	5	6	7	8	9	10
Ratio: SA/V	6	3	2	1.5	1.2	1	0.9	0.8	0.7	0.6

Interpret the Solution: Applying the mathematical understanding or logic from the previous modeling phase to the Children in Hot Cars problem suggests that children (smaller people) will have a higher surface area/volume ratio than adults (larger people). Therefore, children will lose more fluid quickly and they are at a higher risk of dehydration.

Validate the Model: Students can use geometric figures including cuboids, cylinders, or spheres to create physical models of children and adults to extend the simple unit cube approach to construct more elaborate representations to justify their conclusions. This provides students the opportunity to construct representations of people (children or adults) of their choice. Connected mathematical standards and practices will require students' knowledge on measurement and geometry.

Report: At this stage students can write a report in the form of a letter following the modeling framework structure to educate the public about the dangers of leaving children in hot cars.

Here, we described a task that was designed to engage students in mathematical modeling incorporating formative assessment approaches. During the iterations in the process of modeling, there are times at which formative assessment occurs—as students engage in mapping between the modeled world and real world, and when teachers support students in developing capability and facility with the iterative process of modeling (Eames, Brady, & Lesh, 2016). In this case, the task provides students the opportunity to create specific types of models, which they can compare, and draw conclusions. The tasks prompts student to make assumptions, explain their thinking, and provide rationale for their reasoning. A task like the Children in Hot Cars problem gives students opportunities to learn components of the mathematical modeling process that can then be applied to more open-ended summative and formative assessment tasks.

A Framework for Assessing Modeling Performance

The role of assessment in mathematical modeling is critical and complex. Because of the iterative nature of modeling, we believe formative assessment provides an effective and efficient approach in assessing modeling tasks. We present an assessment framework developed for assessing students' modeling performance that take into account the whole modeling process, phases in the modeling process, and the assessment triangle model. Because of the complex nature of mathematical modeling tasks, the assessment framework does not involve rubric scores but suggests two approaches in measuring students modeling performance. The approaches are (a) holistic approach, which involves technicality, originality and management, and presentation (Hall, 1984) and (b) the different phases of the modeling process.

These two methods reflect the elements of the assessment triangle model, and depict both formative and summative form of assessments. Moreover, teachers' minimal guidance, targeting areas that need work, and giving students feedback when working on a modeling task characterize formative assessment of modeling performance (Blum & Borromeo Ferri, 2009). The assessment framework helps teachers evaluate students' modeling performance and promotes familiarity with the elements in the modeling process to both teachers and students. Table 2 provides an explicit description of the assessment framework, which connects to each of the phases in the modeling process and elements in the assessment triangle model.

Table 2. A Framework for Assessing Mathematical Modeling Performance

A Framework for Evaluating Students' Mathematical Modeling Performance		
Categories	**Phase(s)**	**Element(s) Under Assessment Triangle**
Technicality		
Make sense of the task	1	Observation
Create realistic and applicable assumptions	2	Interpretation
Generate the appropriate mathematical model based on assumptions or choices	2 and 3	Interpretation and Cognition
Ability to manipulate the mathematical model to achieve desired results	3 and 4	Cognition
Originality & Management		
Capability to identify situation and formulate problem	1 and 3	Observation and Cognition
Ability to use other techniques or resources including technology	1 and 4	Observation and Cognition
Knowledge of when to change a model, method, or purpose when discussing the problem	6	Interpretation
Ability to recognize what constitute a solution to the real world problem	5 and 6	Interpretation
Ability to work effectively in a group	1–7	Observation, Interpretation, and Cognition
Presentation		
Ability to represent and interpret solution	7	Interpretation
Ability to translate and reflect upon the information	7	Interpretation
Ability to communicate evidently, especially both verbally and in writing	7	Interpretation

Implications and Conclusion

Assessment of mathematical modeling is an emerging area of research, theory, and practice that requires more study with the implementation of the Common Core standards for mathematics and the GAIMME report. Developing additional tools and ideas to measure mathematical modeling performance is timely and necessary. We hope that the ideas presented here will move this work forward. We recommend that other tools or rubrics to assess mathematical modeling be considered and employed depending on the purpose of the assessment (e.g., Anhalt & Cortez 2015; Hall, 1984; Leong, 2012; Munakata, 2006).

When teachers actively engage in formative assessment of modeling tasks during their observation, their interpretations of student models can inform ways to improve instruction, implements strategies, and even revise course content (Diefes-Dux, Hjalmarson, & Zawojewski, 2013). The assessment framework presented here suggests that it is beneficial to assess student mathematical modeling performance not only holistically (summative assessment), but also using task to focus on the phases of the modeling process giving students feedback (formative assessment). We hope that teachers, students, and researchers benefit from the framework presented. With this framework, we hope to advance the classroom assessment of mathematical modeling tasks.

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