Adaptive Learning Systems in Mathematics Classrooms

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Introduction

Preparing the ground for a process of identifying grand challenges in mathematics education, Stephan et al. (2015) collected survey data from a wide range of relevant interest groups. They identified three main themes in the responses. The first of these was the challenge to change perceptions about what it is to do mathematics: "Respondents alluded to the challenge of helping people see that doing mathematics is about problem solving, reasoning, curiosity, and enjoyment, and not about following procedures to get 'the answer' or just about doing well on a test." (ibid., p. 139).

The educational trends of the latest three decades have shifted focus towards social aspects of learning and towards the pupil as an active participant engaged in explorative activities. Mathematics educators have been important contributors in this respect, as stereotypical views of mathematics and its practices seem particularly resistant against change. Devlin (2000) argues that true mathematical activity is motivating by nature as it connects the ubiquitous human capabilities of intellectualizing and socializing. Ricks (2009) elaborates on this view as he writes that "mathematical activity welds together the intellectual and social dimensions of human beings as they collaboratively wrestle with and jointly create mathematical terrain in a process of social mathematizing" (ibid., p. 8). Devlin and Ricks thus exemplify a view of mathematics as a social endeavor characterized by creative exploration, contradicting the stereotypical views of mathematics.

Educators worldwide are currently introducing "adaptive learning systems" in many mathematics classrooms. These computer programs create individualized learning experiences that include tailored learning material and frequent testing. They utilize information from these tests to select and present learning material adapted to the individual pupil's needs (Oxman & Wong, 2014). The rationale behind adaptive learning systems is evident: The teacher cannot help all pupils in a classroom at once. If a computer system can identify a pupil's achievement level, it can provide the pupil with targeted learning material and tasks. Considering the rapid technological development and current implementation rate, most mathematics educators will be acquainted with adaptive learning systems within few years.

In this article, we describe main characteristics of these computer systems and discuss their potential influence on pupils' views on mathematics. We will draw on Schoenfeld (1992) in describing the epistemological trends noted in the first two paragraphs and argue that the adaptive learning systems promote a view of mathematics that differs greatly from these.

Motivation and Aims

We investigated the development and implementation of a specific adaptive learning system from 2014 to 2017. In our experience, discussions about adaptive learning systems mainly evolved around issues of instrumental efficiency: Is this a more time- and cost-efficient way to develop mathematical skills than using existing approaches? Considering our own reception of the adaptive learning systems, we did not investigate critically the epistemological implications of these. Our current critical viewpoint developed gradually through interviews with teachers, group interviews with pupils and observations of pupils using the adaptive learning system.

The discussion in this article is not a new one. Researchers and practitioners have discussed computerized learning and its implications of such since the 60's and 70's. The reason we re-introduce this discussion is the current implementation rate in the Western world. We are most likely entering a time where a substantial share of Western pupils will be using adaptive learning systems in mathematics classrooms. Conclusively, the open problem we address in this article is the disproportionate relation between the high implementation rate on one side and the limited engagement in discussions about its implications on the other side.

Thus, we hope to achieve the following aims in this article:

- Aim 1: To familiarize mathematics educators with adaptive learning systems.
- Aim 2: To identify discrepancies between influential views on mathematical knowledge and the views fostered by adaptive learning systems.

Through the treatment of these aims, we seek to provide mathematics educators with important perspectives that applies not only to adaptive learning systems, but also to the general stream of educational technology. Indeed, the current advances in educational technological will result in many high-quality tools and learning activities. We believe this article will enhance mathematics educators' critical investigation of "the next big thing" that keeps coming along.

We will treat Aim 1 immediately as part of this introduction. After a presentation of the context and data material, we turn to Aim 2. We draw on Schoenfeld (1992) and others to present influential views on mathematical knowledge and discuss how adaptive learning systems relate to these ideas. Note that most developers of adaptive learning systems refer to theories of learning and not to theories of knowledge and knowing. Indeed, we do not want to make an unwarranted comparison between theories stemming from such different paradigms and, specifically, we will not engage in discussions about the efficiency of ALS learning in this article. We turn our attention towards the potential epistemological implications of the current generation of adaptive learning systems.

Adaptive Learning Systems (ALS)

Describing innovations in educational technology is like shooting at a moving target. Articles concerning these technologies are quickly outdated. Thus, our introduction to adaptive learning systems will draw on Oxman and Wong's (2014) rather general introduction. Their presentation encompasses many of the directions in which these systems are currently developing. In the remainder of this article, we will use "ALS" as an abbreviation of an "adaptive learning system". To increase readability, we will repeat the full term periodically.

Oxman and Wong (2014) provide an overview over basic characteristics of adaptive learning systems and important differences between such. In defining an ALS, they refer to the Office of Educational Technology in the U.S. Department of Education (2013):

> *Digital learning systems are considered adaptive when they can dynamically change to better suit the learning in response to information collected during the course of learning. (…) Adaptive learning systems use information gained as the learner works with them to vary such features as the way a concept is represented, its difficulty, the sequencing of problems or tasks, and the nature of hints and feedback provided.* (ibid., p. 27).

In many cases, the pupil will view a learning video before answering a set of question regarding that topic. The ALS will analyse these responses and either 1) consider that the pupil has achieved the foregoing learning goals thus present succeeding learning material to the pupil or 2) analyse the pupil's mistakes and present foregoing learning material according to the identified knowledge gaps. Consequently, a group of pupils using an ALS will quickly find themselves doing different tasks and spending time on different and personally adapted learning material.

Oxman and Wong presents three core elements of an ALS, namely, 1) the content model, 2) the learner model and 3) the instructional model. *The content model* refers to how the ALS organize and structure the mathematical content, at which points the ALS assesses the pupils and the different paths it directs the pupils in after assessment. Some systems have frequent and fine-meshed assessments while other systems deliver greater portions of content before more general assessments. Drawing on the pupil's responses, *the learner model* refers to how the ALS keeps track of the pupil's achievement level. Some systems might have a linear approach where the pupil receives an overall score which determines the further learning path. Other systems might keep track of the pupil's achievements at a sub-topic level and adapt the learning path accordingly.

The instructional model refers to the pedagogical principles underlying the decisions made by the ALS on the pupil's behalf. This is where the ALS combine information from the content model and the learner model and decide on which learning material to present when, either drawing on the accumulated or the most recent information about the pupil. The most sophisticated adaptive learning systems may not only provide successful learners with successive material. These systems may reduce the number of examples and explanations to the highest achievers, "minimizing the number of examples at each step, while guaranteeing that the model produced does not differ significantly from the one that would be obtained with infinite data" (Zliobaite et al., 2012, p. 49).

Most current adaptive learning systems are what Oxman and Wong classify as rule-based. In short, this is when the instructional model contains a series of if-then commands. The complexity varies greatly between systems. The simplest rule-based systems provide the pupil with an independent score after each assessment resulting in either "progress" or "take one step back". The complex systems involve results from several assessments and may result in a variety of outcomes, being different hints, repeated content or new content on the same material. The other class of systems are the algorithm-based systems. These are complex systems demanding great computational power. The ALS will develop the content model and learner model continuously, pairing them efficiently. Algorithm-based systems involve "data mining and advanced analytics to deal with big data, and employ complex algorithms for predicting probabilities of a particular student being successful based on particular content" (Oxman & Wong, 2014, p. 17).

Finally, we present Oxman and Wong's classification of the different degrees of adaptivity between different versions of an ALS. Some are minimally adaptive, some are adaptive at the assessment level and some are adaptive at both the assessment and the content level. A minimally adaptive ALS is a system that provides the pupil with specific responses targeting the specific mistakes the pupil did and refer the pupil to material to review. An ALS is adaptive at the assessment level if it in addition to the properties of a minimally adaptive ALS also uses previous assessments and ensures that the pupil does not progress until all necessary content is learned. An ALS that is adaptive at both the assessment and the content level has the properties of both foregoing systems. In addition, such an ALS will create an individualized learning path that "includes content that statistically has been shown to be most effective at filling in the knowledge gaps identified by the quiz" (Oxman & Wong, 2014, p. 28). The pupil will later receive a new assessment to ensure that the learning path has been successful.

Context and Data Material

Even though the aims of this article are primarily conceptual, the discussion draws on experiences made in the investigation of a specific adaptive learning system in the period from 2014 to 2017. We observed the development process from the very beginning at the drawing board in 2014 through a small pilot study in 2016 and a second pilot study in 2017.

During spring 2014, an educational technology enterprise, a local school community and the authors formalized a three-part collaboration for a governmental funded research-and-development project. The educational technology enterprise would be responsible for development of the adaptive learning system, the local school community would provide schools to the pilot studies and the authors, representing an independent research institute, would document the development and use of the innovation. The content of the adaptive learning system was analysis of functions and curves, which is part of the 11th grade mathematics curriculum in Norway. The target group was 11th grade pupils who had chosen the mathematics course which is most theoretically loaded. The pilot study in 2016 included two schools and the pilot study in 2017 included three schools, including one of the schools from the 2016 study.

The project group intended for the schools to apply the adaptive learning system in a majority of the lessons during a 2–4 week test period. Even though some classes used the adaptive learning system less frequently than the project group planned for, we learned significantly from these pilot studies. The pilot in 2016 mostly regarded technical and administrative processes and data from this is not very relevant in this article. In the 2017 pilot study, we focused on classroom use and outcome of this. The discussion in this article will mainly draw on experiences from the 2017 data collections.

Table 1: Data Material Collected in the 2017 Pilot Study.

School (#classes visited)	Classroom observations	Teacher interviews	Pupil interviews (4-6 pupils/group)
School A (4)	4	4	4
School B (3)	6	3	5
School C (4)	2	5	4
Total (11)	**12**	**12**	**13**

We developed an observational guide that we applied in our classroom observations. This guide consisted of three main parts: 1) Descriptive information about persons and learning activities. 2) Our interpretations of the pupils' responses to this introduction and their engagement with the adaptive learning system during class. 3) Open space to write ideas, considerations and illustrative events that emerged during the observations. Both authors were present in the first few classroom observations in order to calibrate our interpretations and use of the observational guide. Immediately after each lesson, the authors met to discuss their observations and first impressions.

We interviewed 12 teachers who had applied the adaptive learning system for at least one lesson. The interview guide was semi-structured and the main topics were their preparations before class, their evaluation of the adaptive learning system based on their current experiences, and their general considerations of strengths and weaknesses of such systems in mathematics education.

We interviews pupil in groups of 4-6, some at the end of a lesson, some immediately after a lesson and some a few weeks after the final lesson. This way, we hoped to detect a variety of pupil experiences and considerations. We selected pupils for group interviews in collaboration with the teachers in order to ensure a spread in gender and achievement level. In addition to the fixed topics of the interview guide, which concerned their considerations regarding the adaptive learning system, we confronted the pupils with some of our observations and encouraged them to elaborate these particular events. This resulted in valuable information and important validation of our interpretations (Robson, 2007).

Theory and Discussion

We treated Aim 1 in the “Adaptive Learning Systems” section. The forthcoming discussion will also provide input related to Aim 1. We will now turn our focus towards Aim 2, where we point to discrepancies between influential views on what constitutes as “mathematical knowledge” and the view fostered by adaptive learning systems. Schoenfeld’s (1992) “Learning to think mathematically” will serve as a starting point to discover important epistemological characteristics of mathematics. This text holds high status in mathematics education and is among the most frequently referred texts in mathematics education. Even though more than 25 years have passed since Schoenfeld wrote this text, it is still highly relevant. Among many examples, we regard Boaler’s (2015) “Mathematical mindsets” as a recent confirmation of the continued importance of these ideas, where she claims that mathematics is about creativity, sense making and communication. Following the presentation influential ideas, we discuss how adaptive learning systems and the activities facilitated by these systems relate to the presented ideas.

Influential Views on Mathematical Knowledge

Schoenfeld on What it means to Know Mathematics

Epistemology regards the nature of knowing and knowledge. Alan Schoenfeld (1992) treated such issues in a chapter of the 1992 handbook of research on mathematics teaching and learning. The chapter, entitled "Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics" has been reprinted several times. We refer to Schoenfeld to justify our interest in these issues here: "Simply put, a teacher's sense of the mathematical enterprise determines the nature of the classroom environment that the teacher creates. That environment, in turn, shapes students' beliefs about the nature of mathematics" (ibid., p. 71).

What, then, does Schoenfeld claim that mathematical knowledge consists of? First, we make a note about the beliefs pupils often hold: Mathematics is a collection of isolated procedures. Mathematics problems have only one correct solution, and there is only one correct way to solve them. You memorize the correct rule presented by the teacher and apply it as fast as you can. Schoenfeld and others (e.g., Lampert, 1990) claim that this understanding of mathematics is what pupils infer from years of mathematics classroom experience. In short, persons' beliefs about mathematics has received much attention (e.g., Forgasz & Leder, 2008; Markovits & Forgasz, 2017) and Schoenfeld's (rather negatively loaded) description from 1992 seem to prevail.

According to Schoenfeld, *knowing mathematics* means:

1. Exploration of patterns
2. Seeking understanding of mathematical structures and underlying relationships
3. Formulating and framing problems
4. Making conjectures
5. Justifying reasoning processes
6. Generalizing mathematical ideas
7. Communicating mathematical ideas

If these activities characterize mathematics classrooms, then "students will have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be memorized" (National Research Council, 1989, p. 84). We will elaborate on three perspectives deduced from this list, namely, mathematics as an explorative activity (e.g., 1 and 2) where pupils have active agency (e.g., 3, 4, 5 and 6) and enact in a social context (e.g., 5 and 7).

Before doing so, we will comment on the role of formulas and procedures in mathematics. The description that follows paint a broad view of the nature of mathematical knowledge. However, this does not exclude formulas and procedures from being "real mathematics", nor does it exclude routine calculations from the set of basic mathematics skills. Schoenfeld includes this in his notion of mathematics, but claims that "a curriculum based on mastering a corpus of mathematical facts and procedures is severely impoverished – in much the same way that an English curriculum would be considered impoverished if it focused largely, if not exclusively, on issues of grammar" (Schoenfeld, 1992, p. 3).

Knowing Mathematics Means to Engage in Exploration

Drawing on e.g. Pólya's "How to solve it" (Pólya, 1945) and "Everybody counts", Schoenfeld (1992) elaborates on the importance of observations, testing, making estimations and guessing in mathematical work. Mathematicians spend much time analyzing problems and engaging in structured explorations before they devote themselves to a solution strategy. Some mathematics educators criticize school mathematics for portraying mathematics as a fixed set of formulas and procedures. Schoenfeld stresses the view of mathematical knowledge as the ability to explore patterns, systems and quantitative phenomena. This kind of knowledge enables pupils to solve problems where the starting points and the desired finishing points vary from context to context. Thus, school mathematics must enable pupils to navigate flexibly in the mathematical landscape and not only practice a set of predefined routes from fixed starting points to fixed finishing points. The landscape analogy stems from Skemp's (1976) definition of instrumental and relational understanding:

> *"The kind of learning which leads to instrumental mathematics consists of the learning of an increasing number of fixed plans, by which pupils can find their way from particular starting points (the data) to required finishing points (the answers to the questions). (…) In contrast, learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans for getting from any starting point within his schema to any finishing point." (Skemp, 1976, p. 25)*

According to Schoenfeld, the process of building up such a conceptual structure involves activities like activity 1 and 2 mentioned in the list above. Mathematicians will recognize creativity and explorative skills as crucial traits in mathematics. They spend little time conducting well-known routine calculations. However, they look for patterns, propose and challenge hypotheses, explore implications of new insights and make connections between previously disconnected areas. While computers do routine calculations, we depend on creative exploration to know what to calculate. Henningsen and Stein (1997) argue that these capacities develop in classrooms where pupils frequently can engage in dynamic mathematical activities and "rich" tasks. Contrasted to the view presented here is the view of mathematics as "a static, structured system of facts, procedures, and concepts" (ibid., p. 524). If mathematical tasks primarily depend on instrumental approaches with "right or wrong", this will contribute to the pupils' sense of what "doing mathematics" is all about (Schoenfeld, 1992).

Knowing Mathematics Means to Develop Mathematical Agency

Schoenfeld argues that teachers must engage pupils in activities where they get to make conjectures, where pupils' own questions get focus, where they develop their reasoning skills and where they get to explore own ideas. Activity 3, 4, 5 and 6 in the list all provide pupils with agency. According to this view, mathematics knowledge is not constrained to a fixed set of questions and tasks "out there" that the pupils need to learn the answers to, but it is a way for individuals to explore and solve challenges they themselves encounter. Returning to the Skemp analogy, school mathematics must not only provide pupils with a list showing possible routes in the landscape. Pupils must experience the excitement, joy and necessity of identifying where they want to go in this landscape. Having pupils formulate own problems and hypotheses, will transform the young girl or boy from a passive recipient to an agent engaged in meaningful activities. They receive responsibility when they have to justify their reasoning. In addition to the motivational aspects of providing pupils with agency and involvement, these experiences will foster a view of mathematical knowledge as something the pupils co-construct and contextualize. In a more recent publication, Schoenfeld (2014) argues that having to contribute in a collaborative learning process with own conjectures and justifying own reasoning to others are decisive elements in order to develop a mathematical identity.

Notably, opposed to routine calculations and applications of formulas presented by the teacher, such activities stimulate metacognitive activities. Schoenfeld writes about self-regulative processes involved in problem solving, namely, "monitoring and assessing progress 'on line', and acting in response to the assessments of on-line progress" (Schoenfeld, 1992, p. 58). He refers to Flavell (1976), who highlights the importance of active monitoring and consequent regulation and orchestration of learning processes. This is important not only to the learning of mathematics, but also to pupils' meta-learning. They need to become agents in their own learning processes. This includes the ability to assess the individual usefulness of activities in terms of learning, continuously to identify weak spots and knowledge gaps, and to organize a set of learning activities aimed at certain learning goals: Pupils need to "learn how to learn".

Knowing Mathematics Means to Engage in Collaboration and Communicate Mathematical Ideas

Schoenfeld concludes his section on this topic by stating that classrooms must be communities where pupils practice mathematical sense making. He argues that collaboration and communication have profound roles in what it constitutes to know mathematics. It is “an inherently social activity” (Schoenfeld, 1992, p. 3). Schoenfeld refers to many influential researchers (e.g. Deborah Ball, Lauren Resnick, Magdalene Lampert, Alba Gonzalez Thompson, etc.) and their writings in the 1980s to support this claim. Moreover, the many references to publications from interest groups in the same years (e.g. Everybody Counts, The NCTM Standards, different mathematics frameworks, etc.) exemplify why we can characterize the 80’s as the starting point for a shift of focus in mathematics education. Specifically, Lerman (2000) characterizes 1988 as the year of “the social turn” in mathematics education, due to the many influential texts published this year. The new theories described how social activities produce meaning, thinking and reasoning, contradicting the perspective of knowledge as “decontextualized mental objects in the minds of individuals” (Lerman, 2000, p. 13).

Schoenfeld goes beyond the view of collaboration as merely a way of learning. He claims that it is through participating in the “community of practice” that people develop an understanding of an enterprise. A community of practice is a “set of relations among persons, activity, and the world, over time and in relation with other tangential and overlapping communities of practices” (Lave & Wenger, 1991, p. 98), and the classroom is such a community. Taking this perspective, learning does not only happen *through* participation, but learning *is* increased participation. Knowing is thus being part of such a community, and knowing well is to have a high degree of participation. Exemplifying this, Tabach and Schwarz (2017) write that small-group work collaboration in mathematics has become “an educational goal rather than a means” (ibid., p. 273). Conclusively, knowing mathematics includes participation in a community of mathematics practitioners.

Implied by the participation in a community is the ability to communicate. Schoenfeld includes communication of mathematical ideas as part of what it means to know mathematics. Indeed, most educators have experienced how attempts to teach have revealed shortcomings in own understanding of the teaching material. Knowing mathematics means being able to communicate mathematical ideas and arguments to others.

Relating Adaptive Learning Systems to Core Views on Mathematical Knowledge

In the foregoing section, we used Schoenfeld (1992) as a starting point as we prestented epistemological views on mathematics that have influenced the mathematics education community for several decades. Knowing mathematics means, among other things, to engage in 1) mathematical exploration as an 2) active agent who 3) contributes in a learning community. We now draw on these three mentioned perspectives as we discuss some main characteristics of adaptive learning systems (ALS). At the end of this section, we make a note on the potential of ALS in mathematics classrooms.

From Exploration to Procedures

Mathematics education literature suggests that teachers provide pupils with opportunities to explore, to propose questions and make conjectures, and to engage in "rich tasks" and open-ended questions. The pupils' learning paths in such environments differ greatly. The activities facilitated by adaptive learning systems have other characteristics. Developers of adaptive learning systems define a content model. This model depicts how the mathematical content appears in the ALS: Which topics link to other topics? In which order do they occur for the different learners? Which possible routes may pupils take between topics? How much content do the ALS present before pupils receive an assessment?

The content model is, indeed, what Henningsen and Stein (1997) described as a static, structured system of facts, procedures, and concepts. Pupils who make their own discoveries and find similarities and associations between different mathematical topics, are unable to pursue this in an ALS, where connections between content are predefined. Drawing on the Skemp "mathematics is a map" analogy, an ALS provides practice in moving along certain routes on the map. However, the minute your starting point is on a street only a block away from your practiced route, you do not know how to navigate, and if you desire to take an experimental route or see if you can go somewhere else, the systems pushes you back onto the predefined track. An ALS does not support this kind of playful experimentation in the quantitative landscape, which is how pupils develop relational understanding. Even though some adaptive learning systems have detailed and fine-meshed content models, a complete decomposition of mathematics into single components with fixed relations to other components, reduce the number of paths pupils can take in the mathematical landscape. The content model defines how deep into the material pupils can go and which routes they should take between topics.

Pupils who make the same mistakes will experience the same learning path. This path contains a large body of closed questions, as this generation of adaptive learning systems are unable to process answers to open-ended and explorative questions. The instructional model draws on pupils' right or wrong answers in an instrumental way, which Schoenfeld (1992) suggested might contribute to pupils' sense of what "doing mathematics" is: pupils learn implicitly that having mathematical knowledge means knowing the single correct answer to a set of predefined questions, and knowing a set of procedures is what knowing mathematics is all about.

From Being Agents to Being Respondents to a Fixed Body of Knowledge

Adaptive learning systems differ in how the instructional model responds to the learner model. Many systems will utilize both the pupils' assessment scores and the learner model. A high quality and sophisticated ALS will develop complex learner models. It identifies not only the overall ability level of the pupil, but it continuously learns about the approaches that seems most successful for the individual pupil.

Still, an ALS will have to rely on a set of predefined learning trajectories (e.g., Simon 1995). This has important implications regarding mathematical agency. Firstly, an ALS cannot facilitate the activities which according Schoenfeld increases agency, e.g. proposing own problems and conjectures, finding own ways of describing phenomena and justifying mathematical reasoning. An ALS cannot respond to creative suggestions or provide pupils with agency as producers of mathematical content. Knowing mathematics in the ALS sense means, as it seems for the pupils, mastering the "naturally given set of challenges". Thus, pupils become respondents to a fixed body of knowledge as opposed to being engaged in activities they have initiated and defined.

Secondly, we can only regard the optimization provided by an ALS as "optimal" within the universe of learning elements implemented in the system. At a certain point in the learning process, a pupil may benefit from input or actions that a computer system cannot implement. Thus, the pupils lose agency not only with respect to what mathematical knowledge consists of, but also with respect to how they come to know mathematics. They are passive recipients of a learning path and a predefined and limited set of learning activities. The pupils are recipients and not agents in the learning process. This side effect of widespread use of ALS relates to what Schoenfeld describe as self-regulative processes. When asked about the main purpose of formal education, people often use the phrase "learning how to learn". Pupils need agency to develop their meta-learning abilities. The main idea of an ALS is to use the learner model to map the pupil's ability level on the pupil's behalf and to use the instructional model to select learning activities on the pupil's behalf. If the ALS is sophisticated, it will also evaluate the success of this learning strategy on the pupils' behalf. In short, an ALS deprives pupils of meta-learning experiences. The pupils do not need to identify own ability level, propose learning strategies or evaluate these.

From Collaboration and Communication to Individual Isolation

Contrary to the social turn in mathematics education, an adaptive learning system (ALS) mainly facilitates individual activities. Pupils equip with a headset and work isolated from the classmates. The instruction model of an ALS depends on accurate input to the learner model in order to provide an adapted learning experience. Thus, any interference from teachers or other pupils is inappropriate. If the pupil receives guidance from other persons, the learner model will be imprecise and consequently the instruction model will not succeed in providing a well-adapted learning path. Pupils confound the adaptivity if they collaborate.

According to the views on mathematical knowledge presented in this article, pupils need to learn to take part in a mathematical discourse and to communicate mathematics efficiently. This generation of adaptive learning systems cannot facilitate any kind of training in communication, as its opportunities to assess pupils' open-ended ideas and explanations are limited. Nor does it facilitate fruitful group discussions and participation in a learning community, as pupils after an ALS session will have been working on different assignments and topics.

This setting, where pupils can complete a full mathematics course alone with headsets, where collaboration confound the learning process and where communication is absent, may influence their view on mathematical knowledge. It is restricted to the achievements you can attain as an isolated individual.

This issue relates to the issue of mathematical agency. The pupils develop their mathematical identities through interaction with other persons, as they contribute in a mathematical discourse and receive direct or indirect feedback from other persons. They receive few identity clues through listening to learning videos, responding to computer assigned tasks and having the responses labelled "correct" or "incorrect". Begin an active mathematical agent depends on a well-established mathematical identity, which develops partly due to meaningful interaction with other persons. In her recent research commentary "Authority, identity, and collaborative mathematics", Langer-Osuna (2017) elaborates on this issue.

The Potential of Adaptive Learning Systems

The above limitations provide an argument for why an ALS cannot take up a substantial amount of the time pupils spend in the mathematics classroom. We argue that this may foster a narrow view of what it means to know mathematics.

Irrespective of this critique, we suggest that an ALS has potential to contribute positively to school mathematics. An ALS facilitates individual activities, but this is both welcome and necessary in some learning contexts. When pupils do homework, we cannot assume that other persons are present for guidance or collaboration. The interactivity and adaption provided by an ALS is preferable to no interaction or adaption at all. Moreover, when a class is practicing for a test or repeating learning material, the pupils may benefit from working individually on the specific topics that they themselves need to repeat. An ALS can be efficient in this respect, providing the pupils with training on topics they do not master sufficiently.

Indeed, mathematics is more than procedural knowledge, but such knowledge is part of an important basis. All pupils need to spend some time practicing basic algorithms. Thus, we consider the potential of adaptive learning systems in homework, in practicing routine skills and in repetition to be promising.

Conclusion and Future Research

Conclusion

In this article, we presented basic characteristics of adaptive learning systems (Aim 1) and discussed the view of mathematical knowledge they may foster (Aim 2). Summarizing our critique, the adaptive learning systems provide mathematical activities where pupils move around in the system according to predetermined patterns unable to explore own insights and ideas. They are not in control of their own learning process and the activities are utterly individual endeavors. Widespread use of such systems may contribute to pupils developing a narrow view of mathematical knowledge as procedural, fixed and individualistic.

Conclusively, the current generation of adaptive learning systems should not occupy much time in mathematics classrooms. We encourage teachers and school leaders planning to use ALS to discuss the following claim: "Widespread use of an ALS in mathematics classrooms will provide pupils with a narrow view of what mathematics is, namely, procedural, static and individualistic." Such a discussion may support a thoughtful implementation of an ALS, for instance when pupils practice procedural skills as homework or before exams.

Future Research

With regard to the epistemological issues raised in this article, we expect future investigations of how adaptive learning systems actually, and not hypothetically, influence pupils' views on mathematics. Will anyone succeed in creating an ALS without the weaknesses elaborated on in this article? Will the next generation of ALS include activities where pupils explore own ideas and make knowledge connections in a learning community? Alternatively, will the limited scope of such systems influence the constant negotiation of the content of school mathematics?

With regard to theories of learning, both teachers, school owners and technology developers are curious about the efficiency of adaptive learning systems. Do the adaption provided by an ALS exceed the value of the existing adaption, where pupils support each other, choose exercises from targeted difficulty levels and spend time on the exercises they answered incorrectly? Where teachers who know their pupil's learning history and preferences can spend a minute or two providing targeted support?

Despite the critique presented in this article, we hope to find adaptive learning systems in many mathematics classrooms in the future. It relies on a collective effort of teachers, ALS developers and mathematics education researchers to ensure the quality of these.

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References

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass.

Devlin, K. J. (2000). *The math gene: How mathematical thinking evolved and why numbers are like gossip.* New York: Basic Books.

Flavell, J. (1976). Metacognitive aspects of problem solving. In L. Resnick (Ed.), *The nature of intelligence* (pp. 231–236). Hillsdale, NJ: Erlbaum.

Forgasz, H. J., & Leder, G. C. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan & T. Wood (Eds.), *International handbook of mathematics teacher education: Vol. 1. Knowledge and beliefs in mathematics teaching and teaching development* (pp. 173–192). Rotterdam/Taipei: Sense Publishers.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524–549.

Lampert, M. (1990). When the problem is not the problem and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29–63.

Langer-Osuna, J. M. (2017). Authority, identity, and collaborative mathematics. *Journal of Research in Mathematics Education, 48*(3), 237–247.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CT: Ablex.

Markovits, Z., & Forgasz, H. J. (2017). “Mathematics is like a lion”: Elementary students’ beliefs about mathematics. *Educational Studies in Mathematics, 96*(1), 49–64.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

Oxman, S., & Wong, W. (2014). *White paper: Adaptive learning systems*. DV X/DeVry Education Group and Integrated Education Solutions.

Pólya, G. (1945). *How to solve it*. Princeton: Princeton University.

Ricks, T. E. (2009). Mathematics is motivating. *The Mathematics Educator, 19*(2), 2–9.

Robson, C. (2007). *Real world research: A resource for social scientists and practitioner-researchers.* Oxford: Blackwell Publishers.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–371). New York: Macmillan.

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational Researcher, 43*, 404–412.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114–145.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teacher, 77*, 20–26.

Stephan, M.L., Chval, K.B., Wanko, J.J., Civil, M., Fish, M.C., Herbel-Eisenmann, B., Konold, C., & Wilkerson, T.L. (2015). Grand challenges and opportunities in mathematics education research. *Journal for Research in Mathematics Education, 46*(2), 134–146.

Tabach, M. & Schwarz, B. B. (2017). Professional development of mathematics teachers toward the facilitation of small-group collaboration. *Educational Studies in Mathematics, 97*(3), 273–298.

U.S. Department of Education, Office of Educational Technology (2013). Expanding evidence approaches for learning in a digital world. Washington D. C.

Zliobaite, I., Bifet, A., Gaber, M., Gabrys, B., Gama, J., Minku, L., & Musial, K. (2012). Next challenges for adaptive learning systems. *SIGKDD Explorations, 14*(1), 48–55.