Dynamic Math Teaching and Learning: Zones of Math Knowledge Creation Framework

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Introduction

Successful math teaching and learning is elusive to many educational systems around the world. Students' math achievement across various jurisdictions is trending downward regardless of the efforts to boost the love and learning of math. The debate that mathematicians, educators, curriculum developers, policy makers, and public are currently engaged in focuses primarily on the pedagogy of math, that is, how should math be taught? From the author's perspective, this debate is fundamentally steeped in a question that has been asked for centuries involving how individuals actually acquire knowledge. Without a doubt, this is an important debate that regularly polarizes the masses between reform and traditional methods of teaching math. The concept of reform math is associated with other terms, for example, discovery-based learning (Destrebecqz, 2004; Rittle-Johnson, 2006), inquiry-based learning (Maaß & Artigue, 2013), constructivist approach (Richardson, 2003) and new math (Kilpatrick, 2012).

Although there is no definitive definition for reform mathematics, consistent principles and beliefs in the literature are used by this author to define reform mathematics. For the purpose of this article, reform math is defined as a dynamic teaching and learning approach, which encompasses the belief that students learn by socially interacting with others to delve into complex and authentic mathematical tasks. Ultimately, this enables learners to build upon current knowledge, construct new knowledge, advance deep conceptual understandings about math ideas, and develop efficacy and interest in the field of math (NCTM, 2000, 2014; Ross, McDougall, & Hogaboam-Gray, 2002).

On the other side of the spectrum, one finds a traditional approach, that is, a transmissive style to teaching and learning math. This approach can be defined as a belief that teachers hold math knowledge and disseminate it to students. Subsequently, instruction is dominated by the teacher requiring students to memorize facts, practice procedures, and gradually learn different ways to solve problems. A typical lesson, based on a transmissive approach, involves whole group review of previous knowledge, teacher demonstration of new knowledge, and student practice to gain new knowledge. Based on these definitions alone, there is a wide divide between reform and traditional approaches to teaching and learning math. Not surprisingly, disparate groups have been formed based on their beliefs, each citing evidence to promote either approach (Ross et al., 2002).

For several decades, the National Council of Teachers of Mathematics (NCTM) has advocated for reform math (NCTM, 1989, 2000, 2014). Regardless of continued efforts to implement reform math, NCTM reports limited progress. The council identifies several challenges that still exist such as heavily relying on learning procedures at the expense of understanding the conceptual foundations of the procedures (NCTM, 2014). In addition to NCTM, other organizations also proclaim the importance of reform math (Ontario Association for Mathematics Education, 2017). For example, the Ontario Association for Mathematics Education (OAME) is a strong advocate of reform math pedagogy. As part of OAME's vision for teaching and learning math, it highlights the importance of building communities of math learners in which students are dynamic participants in inquiry. Students are not merely recipients of knowledge, but rather they actively create math knowledge in a social context by asking questions and exploring their understandings (OAME, 2017). The social construction of knowledge is foundational to a dynamic approach to teaching and learning math, as well as a tenet of reform principles. Without question, past, current, and future efforts to promote knowledge creation in math classrooms is at a contentious crossroad, which presents opportunities and pitfalls for policy makers and teachers.

As jurisdictions around the world continue to investigate opportunities to promote competencies in math, the *Zones of Math Knowledge Creation Framework* is presented in this paper for consideration and further research. The foundational underpinning of this framework offers a perspective of why many educational districts have not actualized the gains in math that were strategically planned. To promote growth in the areas of student understanding and appreciation of math, teachers' personal math identities (PMIs), as well as their teaching and learning approaches implemented in classrooms must be contemplated and analyzed. When teachers' PMI increases and their willingness to engage in dynamic teaching and learning approaches advances, an *Optimal Zone* of math knowledge creation can be attained by individuals and ultimately by educational systems.

Although the framework introduced in this paper is not jurisdictionally dependent, Ontario's efforts to support math instruction in classrooms over a decade and a half are explored to offer readers a context. The importance of math to policy makers was highlighted in 2004 by the introduction of Ontario's Literacy and Numeracy Secretariat (LNS). Although literacy and numeracy were espoused as being equal priorities provincially, teachers questioned the focus on math compared to literacy … "There is such a focus on literacy and such support for literacy, numeracy is on the back burner" (Canadian Language and Literacy Research Network, 2009, p. 63).

In 2016, math was again identified as a priority in the province through the Renewed Math Strategy (RMS), which included requirements such as sixty minutes of protected math learning time each day in elementary classrooms, as well as resources for both elementary and secondary systems, e.g., math lead teachers, professional learning opportunities for teachers and principals (Zegarac, 2016). Improving student achievement in the area of math was again identified as a priority by the Government of Ontario in 2017, as there was a commitment to review policies involving curriculum and assessment to ensure that students acquire the transferable skills necessary for the future workforce (Office of the Premier, 2017).

The focus on math became a priority when Ontario's provincial and international assessments demonstrated insignificant gains and actually declined over the years in some cases. Ontario assessments in math are administered through the Education Quality and Accountability Office (EQAO) at grades 3, 6, and 9. Over a span of a decade (2009–2018), results of Ontario students achieving the provincial standard dropped by 14 percentage points (63% to 49%) in grade 6 and 9 percentage points (70% to 61%) in grade 3 (Education, Quality and Accountability Office, 2018a).

At the secondary level, two assessments are provided in grade 9 based on the course of study, applied or academic. In general, students enrolled in applied courses are primarily destined for college, i.e., career-orientated institutions offering diplomas or certificates, whereas students in academic courses are most likely to have a university pathway, i.e., degree granting institutions (Reid & Reid, 2017; James & Turner, 2017). The provincial results for applied and academic assessments remained fairly static during this same time. The percentage of students achieving the provincial standard in the applied level courses is in the mid 40s, while percentage of students achieving the provincial standard in academic math courses is in the mid 80s (EQAO, 2018b). The significant achievement gap between the two courses is of concern for a variety of reasons. One major issue is the overrepresentation of students in applied courses whom are from equity-seeking groups, including: low socio-economic status communities (Hamlin & Cameron, 2015), Black students (James & Turner, 2017), and students with special education needs (EQAO, 2018b).

Ontario is currently in the midst of proclaiming a *back to the basics* approach to teaching and learning math. In Ontario's Plan for the People (Fedeli, 2018), the government claims a need to end discovery math (i.e., reform math) due to the decline in provincial large-scale assessment scores … "A discovery-based learning environment does not teach students the fundamentals of basic math" (p. 78). However, large-scale Ontario assessment results have clearly identified that grade 3 and 6 students demonstrate higher achievement in the area of basic math skills requiring simple recall of content knowledge. These same students have more difficulty with math that requires them to apply their knowledge or demonstrate critical thinking (EQAO, 2019). With forthcoming changes to Ontario's math curriculum, along with new teacher education certification requirements in the area of math proficiency (Ontario Ministry of Education, 2019), the landscape of math education in this province is poised to dramatically change for years to come.

Literature Review

Many teachers tend to equate some of the factual aspects of math with a need to engage students in repetitive practice without conceptual understanding of the computations themselves (Boaler, 2019). Bruce (2007) considered why a traditional model of teaching math that "focused on basic computational procedures" continues to be commonplace in classrooms (p. 2). She purports that the greatest challenge many teachers face when embracing reform practices is that most teachers never experienced this model of teaching as students themselves (Bruce, 2005). Researchers also suggest that the absence of reform math is resultant from teachers' lack of content knowledge within this subject area (Reid & Reid, 2017; Ponte & Chapman, 2008). "Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content" (Ball et al., 2008, p. 404). As any future state of math teaching and learning is envisioned, it is necessary to explore the following areas of theory and research: reform and traditional models of math instruction, math knowledge, math identity, and knowledge creation.

Reform and Traditional Models of Math Instruction

At the core of reform math is a focus on understanding math at a conceptual level through problem-solving opportunities that involve connections across math ideas. This constructivist approach to knowledge creation has led to changes in curriculum and textbooks, as well as fueled debate regarding how to best teach math (Baker et al., 2010). In fact, this debate is often referred to as the *Math Wars* (Ross et al., 2002; Schoenfeld, 2004). As with most debates, the positions are juxtaposed and identified as dialectics (White-Fredette, 2010). The counterpoint to reform math is a traditional approach to instruction. In this transmissive model, the learning of math is viewed as highly sequential and achieved when the teacher transmits knowledge to their students in order to be practiced and memorized. This rote learning approach does little in the way of allowing students to undertake problems in innovative ways, nor apply skills to unfamiliar problems (Baker et al., 2010; Mann, 2006). Regardless of the philosophical conceptions of how to teach math, any attempts to change instructional practice on a large-scale will necessitate professional learning that is collaborative and connected to teachers' classrooms (Bruce, Esmonde, Ross, Dookie, & Beatty, 2010).

Many researchers argue that, despite attempts to support change in math instruction over the years, envisioned transformation has been largely missing (Haeck, Lefebvre, & Merrigan, 2014; Hiebert et al., 2005; Marshall, 2006). Studies illustrate how reform recommendations are regularly executed superficially or abandoned altogether (Ross et al., 2002). Charalambous and Philippou (2010) suggest that teachers who are comfortable in teaching math through transmissive methods often have more concerns about reform math. Specifically, teachers may lack confidence in their abilities to successfully implement complex teaching practices, as well as fear the potential of creating complicated learning situations for students who often struggle in math.

Moreover, when implementing instructional practices that are unfamiliar, Spillane (2005) gives evidence that teachers are less likely to reach out to their peers for support in math than in literacy. Wide sweeping transformation is complex and demanding for any system. However, for reform math, the challenges are heightened due in part to teachers' lack of content knowledge, fears of the unknown impacts on students, as well as fears of being vulnerable to one's peers.

Math Knowledge

For teachers to effectively create dynamic math teaching and learning environments, it requires a robust knowledge of math. Researchers have classified different types of knowledge that teachers must possess to successfully teach math (Ball et al., 2008). These types of knowledge include math knowledge for teaching (MKT), as well as a subset called math content knowledge (MCK). MCK is defined as the basic math knowledge required for an individual to be regarded as mathematically literate in society (Reid & Reid, 2017). Without MCK, teachers are often unable to make informed instructional decisions based on the needs of students (Lui & Bonner, 2016).

Additionally, teachers require deep conceptual understandings to implement effective teaching and learning strategies in support of students' learning of math (Ball et al., 2008; Ma, 1999; Thames & Ball, 2010). For Ball et al. (2008), the conceptualization of MKT involves teachers' knowledge of basic math content, conceptual understanding of the content, understanding of their own students and the math curriculum, as well as a math pedagogy. Silverman and Thompson (2008) also propose a framework for envisioning MKT in which teachers must possess a deep understanding of math ideas. This conceptual knowledge enables them to anticipate how students might understand and think about the ideas and design learning opportunities for students to develop math ideas and make connections to other ideas.

Researchers have signalled the importance of conceptual understanding of math to help students make conceptually-based connections and support the understanding of algorithms (Reid & Reid, 2017; Boaler, 2019; Ponte & Chapman, 2008; Thames & Ball, 2010), as well as make instructional decisions in the moment based on student interactions with the concepts (Mason & Davis, 2013). Therefore, a strong foundation of MCK is essential for teachers to develop the necessary skillset to support students in developing conceptual understandings, facilitate math discourse, and promote problem solving (Reid & Reid, 2017; Reid, Reid, & Hewitt, 2018; Hill et al., 2016). Without a concerted effort to develop MCK, attempts to build the capacities of teacher's MKT are undoubtedly compromised.

The relationship between procedural and conceptual understanding plays a significant role as teachers work to develop their own MCK and MKT (Boaler, 2019; Hill & Ball, 2009). Procedural knowledge is defined as understanding how to employ an array of actions, rules, or algorithms to answer certain types of math questions (Hiebert, 1992; McCormick, 1997). All too often, teachers' personal experiences as math students involve an over reliance on algorithms (Reid & Reid, 2017; Bruce, 2005; Lui & Bonner, 2016). This dependency has been observed in preservice teachers with low levels of MCK (Thanheiser et al., 2014).

There is no doubt that algorithms offer efficiencies to quickly carry out computations, however, the algorithms are often not easily understood intuitively, and at times counterintuitive (Bartell, Webel, Bowen & Dyson, 2012; Philipp, 2008). Developing procedural proficiency alone, without a deeper understanding of why the procedures are followed, presents issues of retention over time (Bransford, Brown, & Cocking, 1999; Hiebert et al., 2003) and discourages students from seeking out the conceptual underpinnings of the math ideas (Hiebert, 1999; Hiebert et al., 1996). Additionally, NCTM (2014) promotes procedural fluency as advanced flexibility of math ideas to deeply understand the appropriate procedures to use to effectively solve problems. This type of fluency develops through problem-solving that fortifies abilities to reason, communicate, and justify, as well as build MCK. Ultimately, dynamic approaches to teaching and learning involves both conceptual understanding and procedural fluency.

Math Identity

The *math identity* of a teacher encompasses how one views their own mathematical abilities. A teacher's math identity is influenced by personal experiences with math learning, how others view their math skills, prior math achievement, content knowledge, levels of math anxiety and efficacy, as well as gender, race, and socioeconomic status. Researchers suggest that strong math identities are associated with higher levels of achievement (Cass, Hazari, Cribbs, & Sonnert, 2011; McGee & Martin, 2011). Regrettably, the divide between strong and weak math identities has long been associated with key sociocultural factors such as gender, race, and socioeconomic status.

Over the years, the field of math has been recognized for its part in sifting and sorting students with long lasting effects. Those who excel at math can access various careers including the science, technology, engineering, and mathematics (STEM) fields; while others find doors to post-secondary STEM pathways closed due to lack of access to senior high school math and science courses (Blickenstaff, 2005; Wang, 2013), or face pressure to enrol in less challenging courses (Smyth & McArdle, 2004). Researchers have highlighted systemic issues such as significant gender and race gaps in STEM fields (National Science Foundation, 2011), beginning with higher percentages of marginalized students dropping out of senior math courses in secondary school (Wei, Lenz, & Blackorby, 2013). Studies have also demonstrated that racialized women are threatened by negative stereotypes, which dissuade them from STEM trajectories, as well as coping with their own low self-efficacy in these fields (Duran, Lopex, & Hughes, 2015; Gunderson et al., 2013). If the field of STEM or even math alone was used as a litmus test for societal equity, results unfortunately suggest a long road ahead to close systemic gaps.

A teacher's math identity is not static as all identities continue to be socially constructed and negotiated (Azmitia, Sye, & Radmacher, 2008). The theory of intersectionality presents insights into the interrelationship of an individual's social identities (Crenshaw, 1989). This intersectionality allows researchers and educators to consider math through the identities of individuals and groups, as well as the ongoing challenges presented due to stereotypes based on race, gender, and socioeconomic status. For instance, females have long underperformed in various math fields, especially in societies where gender parity has not made gains (Else-Quest, Hyde, & Linn, 2010; Guiso, Monte, Sapienza, & Zingales, 2008). Of most importance is that people live through one or more intersecting stereotypes. These stereotypes exist in one's own beliefs or the beliefs of others.

The fear of confirming a negative stereotype through action or personal feature has been recognized as stereotype threat (Steele 1992, 1997). These stereotypes include beliefs that certain racial groups are not suited for the field of math (Steele & Aronson, 1995; Tine & Gotlieb, 2013); girls are not predisposed to be as good at math as boys (Tomasetto, Alparone, & Cadinu, 2011); or those from low-socioeconomic environments are not equipped to succeed in higher levels of math (Tine & Gotlieb, 2013). Adverse effects on math performance are further compounded when negative stereotypes intersect (Brown & Leaper, 2010; Tine & Gotlieb, 2013).

Another threat to one's math identity involves the model minority myth, that is, the belief that certain races are better positioned for success. As a group, Asians are commonly perceived as being proficient in math and science (Lee, 1996; Wong & Halgin, 2006). At first glance, this might appear to have positive ramifications, however, the model minority myth can result in fears of not living up to the myth and ultimately lowering math and science outcomes (Cheryan & Bodenhausen, 2000). It quickly becomes evident that the intersectionality of one's math identity is not only complex, it is consistently reinforced or challenged by societal beliefs and expectations.

The constructs of math efficacy and math anxiety influence how math is conceptualized, approached, and taught, each influencing one's math identity. Bandura's (1977, 1986, 1997) research on teacher efficacy is founded on the theory of social learning. As part of self-efficacy, teacher efficacy is essentially the personal conceptions of one's own teaching abilities (Enochs, Smith, & Huinker, 2000). Teacher efficacy in the area of math can influence teachers' efforts in the classroom, perceived impact on student learning, reactions to disappointment, and how stress is encountered (Swackhamer, Koellner, Basile, & Kimbrough, 2009). The promotion of teacher efficacy is correlated with instructional choices and readiness to attempt new strategies (Swars, Daane, & Giesen, 2006).

Positive influences on instructional practices in the classroom also attribute to decreases in math anxiety in teachers (Vinson, 2001), as well as math achievement (Beilock, Gunderson, Ramirez, & Levine, 2010). According to Vinson (2001), math anxiety is an emotional feeling of nervousness that individuals may possess about their limited understanding of math. High levels of math anxiety influence the instructional choices that teachers make in the classroom, most often emulating transmissive approaches such as high levels of lecture style teaching, reliance on textbooks, memorization of rules, practice of basic skills, and low-level worksheets instead of rich problem-solving activities (Finlayson, 2014). Moreover, teachers with anxiety connected to teaching math are correlated with lower math achievement in students (Hadley & Dorward, 2011). This interplay between math anxiety, efficacy, and instructional choices may further amplify a vicious cycle observed in math education, one in which dynamic teaching and learning practices are superseded by transmissive teaching approaches (Reid & Reid, 2017).

Knowledge Creation

Knowledge creation is defined as the process of interpreting information based on the context and understandings of individuals and groups for the action of mind or body (Nonaka & Takeuchi, 1995). This process of creating knowledge is a foundational component of the social experiences that occur in dynamic math teaching and learning environments. Social learning happens internally within the individual as well as by observing and interacting with others (Bandura, 1962). A conscious awareness can develop in an individual about how learning occurs (Derry & Murphy, 1986; White & Mitchell, 1994), a process called metacognition, which includes *learning how to learn* (Smith, 1982) or *cognition about cognition* (Flavel, 1985).

In Ontario, metacognition is generally defined as students being able to monitor their own learning and develop a capacity to reflect on classroom experiences to support continued learning. Metacognition via reflections on personal thought processes through problem solving and investigations is one of seven mathematical processes expected in every elementary and secondary math classroom (OME, 2005a, 2005b). Notably, a reform approach to teaching and learning is woven throughout the Ontario math curriculums, which promotes students to use their knowledge, think critically, and nurture an enjoyment in math. In alignment with NCTM's (2014) reform approach, the Ontario curriculum also prioritizes students' reflection on thinking, their own and that of others, in order to create math knowledge that is internalized and deeply conceptualized.

When examining learning in math classrooms, knowledge creation is realized when students embrace productive struggle in which they work on problems that are not easily solved (Hiebert & Grouws, 2007; Warshauer, 2014). Further, students engage in tasks that encourage varied and purposeful discourse (Hufferd-Ackles, Fuson, & Gamoran-Sherin, 2004; Lester, 2007) and problems with multiple entry points and solution strategies in order to develop flexibility in understanding of math ideas (Leikin, 2013; Rittle-Johnson & Star, 2009).

The role of the teacher involves decisions that are made continually, and these decisions rely on a knowledge base developed over years within and beyond the educational environment. Polanyi (1962) identifies these two forms of knowledge as tacit and explicit. Tacit knowledge includes knowledge that individuals rely on daily but have a difficult time explaining definitively. This internal knowledge allows an individual to perform many tasks without consciously thinking about the specific actions taken. Alternatively, explicit knowledge is identified as knowledge that is easily externalized and shared with others without difficulty. As teachers gain experience in facilitating the learning of math, teachers' minds are available for deeper thinking about misconceptions that students might encounter, how students might answer a particular problem, as well as making connections between math concepts, representations, and procedures. If teachers' tacit knowledge is based on limited MCK, including their own misconceptions or inaccurate understandings, these errors can negatively affect decision-making and instructional interactions with students. It therefore becomes critical for teachers to consistently reflect on their understandings and practices, exploring explicit knowledge and exposing tacit knowledge in order to create new knowledge for math teaching.

Zones of Math Knowledge Creation Framework

The *Zones of Math Knowledge Creation Framework* includes approaches to math teaching and learning while connecting to teachers' personal math identities (see Figure 1). Deconstruction of the framework unveils teachers' specific strengths and needs so professional learning opportunities can be tailored for improved MCK and MKT. The four zones take into account various constructs: reform and traditional math instruction, math knowledge, math identity, and knowledge creation. As such, the *Zones of Math Knowledge Creation Framework* illuminates how knowledge creation materializes in math classrooms. Teachers bring to class each day their unique personal experiences with math, beliefs about their math abilities, levels of efficacy, anxiety towards math, as well as sociocultural factors. These important elements influence one's personal math identity (PMI).

As teachers plan their instructional approaches for math teaching and learning, PMI must be considered. A "one size fits all" approach to professional learning in math teaching will not be effective due to the unique PMI experiences teachers bring to the classroom. Hence, it would be unreasonable to expect each and every teacher to implement uniform instructional strategies without understanding teachers' current PMI. In fact, one size fits all approaches can be counter-productive in which outcomes could involve increased levels of math anxiety and superficial implementation of instructional practices.

Dynamic teaching and learning instruction provide environments for students to socially learn with one another. Knowledge creation is actualized when students engage in challenging and authentic mathematical tasks in efforts to reinforce their knowledge base, construct new knowledge, advance deep conceptual understandings, and develop efficacy and interest in math. In attempts to promote the *Optimal Zone* of math knowledge creation, a strategic, differentiated, and long-term approach to supporting the professional learning of teachers is necessary. To broadly consolidate this zone, large- and small-scale knowledge mobilization strategies are necessary to foster cultures of inquiry, trust, risk-taking, and learning (Reid, 2015).

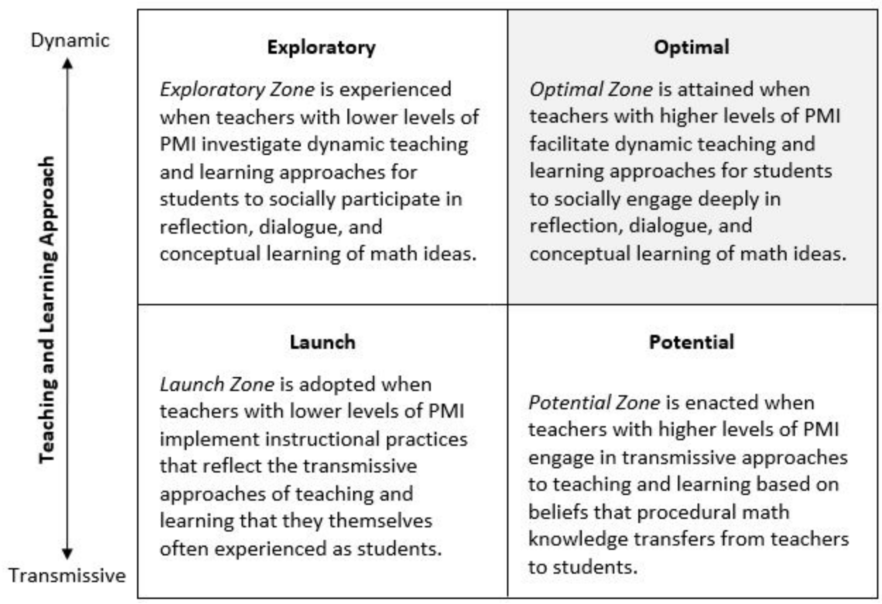

Figure 1. Zones of Math Knowledge Creation Framework

Figure 1. The Zones of Math Knowledge Creation Framework provides an opportunity to view teachers' current approach to teaching and learning math, along with their personal math identity (PMI). As teachers identify their location within the framework, plans can be developed to move toward higher PMI and more dynamic approaches to teaching and learning. In doing so, teachers can advance toward the Optimal Zone of math knowledge creation.

To conceptualize the *Zones of Math Knowledge Creation Framework*, the author envisioned two continuums: 1) teaching and learning approach situated on the vertical axis and 2) personal math identify (PMI) situated on the horizontal axis. The teaching and learning math continuum spans between two approaches: transmissive (i.e., traditional) and dynamic (i.e., reform). As emphasised previously, a highly transmissive style of instruction involves a focus on a lecture style of teaching, procedural learning, and rule memorization. Whereas a dynamic approach to instruction engages students in rich problem-solving activities, with a focus on conceptually understanding math ideas.

It is critical to note that dynamic approaches to teaching and learning math includes foci such as automaticity of basic facts, learning of procedures, as well as practice with algorithms. The essential difference between the approaches is the requirement of conceptual understanding in a dynamic environment. Therefore, how math activities are regularly facilitated differ based on the teaching and learning approach implemented in the classroom.

The horizontal continuum presents the construct of PMI, how a teacher views their mathematical abilities. Based on various factors that would influence their PMI (e.g., math experiences, personal beliefs about math ability, efficacy, anxiety, sociocultural), an individual would situate themselves on the PMI axis. For example, individuals with negative experiences in math as students, high levels of math anxiety, and exposed to stereotype threat, would likely place themselves near the far-left end of low PMI. Whereas individuals with high levels of efficacy and content knowledge in math look to the far-right end of the axis indicating high PMI. The placement on this axis can change based on teachers' contexts such as specific math concepts, grade levels, or external pressures to engage in targeted approaches to teaching and learning math. The goal for all teachers is to move into the *Optimal Zone*. To systemically support teachers to move toward this preferred zone as professionals, each of the four zones must be understood.

Optimal Zone

To further math knowledge creation, the *Optimal Zone* – situated in the upper right quadrant – is the proposed instructional goal for all individuals, classrooms, and learning organizations. When teachers identify higher levels of PMI due to confidence and knowledge of math as a student and teacher, combined with a willingness to explore dynamic approaches to teaching and learning, the highest levels of math knowledge creation for students and teachers occur. In this quadrant, teachers have the knowledge foundations in math required and they also work in a culture that affords them to continually engage their students in reflection, dialogue, and conceptual learning of math ideas.

In order for a teacher to move to this zone of the highest level of knowledge creation, professional learning and supportive networking environments are essential. Within these supportive cultures, teachers can access networks when questions arise or when learning activities do not evolve as planned. Then, teachers can collaboratively investigate students' capacities, understandings, and needs, all of which are drivers for teacher engagement (Little, Gearhart, Curry, & Kafka, 2003; Perry & Lewis, 2010). It is essential for teachers to encounter sustained opportunities to explore dynamic, creative, and innovative instructional approaches in efforts to deeply understand students' interactions and relationships with math. As teachers engage in more dynamic teaching and learning approaches, their PMI can also improve, thereby moving toward the upper right quadrant of the framework.

Launch Zone

Teachers with lower levels of PMI who implement instructional practices that reflect a more transmissive approach to teaching and learning are situated in the *Launch Zone* – found in the lower left quadrant of the framework. Here, teachers may find comfort in math practices that they encountered as students during elementary, secondary school and/or a teacher preparation degree. Through transmissive approaches to teaching, students are rarely engaged in developing their conceptual understandings of math ideas (NCTM, 2014). Furthermore, teachers may bring their own math anxiety to the classroom, which can negatively impact students learning.

Unfortunately, when teachers are anxious about math instruction, they are prone to avoidance behaviours by spending less time planning lessons and engaging in math instruction itself (Hembree, 1990; Trice & Ogden 1986). Researchers have also found that teachers with negative math attitudes or less efficacy toward teaching math are more likely to use transmissive approaches to instruction and avoid dynamic and innovative approaches (Karp, 1991). As teachers continue to evolve as professionals, they can move along both continuums toward higher levels of PMI and more dynamic approaches to teaching and learning. These positive shifts can occur by further developing MCK, efficacy, and MKT, as well as challenging sociocultural factors.

Exploratory Zone

The *Exploratory Zone* – situated in the upper left quadrant – incorporates teachers who are developing a higher level of comfort for increasing dynamic approaches to teaching and learning math, as well increasing their PMI. For this to occur, dynamic teaching and learning approaches are best explored within a supportive learning community. This professional assistance is essential as teachers' PMI is presently lower on the continuum. Through mentoring, professional learning, and collaborative inquiry, teachers could find themselves in positions in which they safely implement dynamic teaching and learning approaches without fears of isolation or reprimand. With successes in the classroom, teachers' PMI can also improve.

One of the risks for systems attempting to move teachers toward this quadrant occurs when innovative approaches to teaching math are implemented superficially. Initiatives to improve math become more about the activities themselves such as group work or using manipulatives instead of developing a deep understanding of math teaching and learning. Rather than merely going through the motions of what seemingly appears to be reform math, teachers should be engaged in investigating the broader understandings of how students effectively learn math concepts and integrating various strategies when most appropriate (Hiebert et al., 2005; Spillane, 2000).

Another risk associated with promoting a systemic move toward this quadrant is realized when teachers feel pressure to enact dynamic approaches in their math classrooms without receiving foundational assistance. In these situations, anxiety can ensue, resulting in a lowered PMI, thereby creating a greater gap to entering the *Optimal Zone*. Although it is important for teachers to explore dynamic instructional approaches, professional learning environments must be constructive and efficacious for knowledge creation to emerge.

Potential Zone

In the *Potential Zone* – situated in the lower right quadrant – teachers have strong MCK and efficacy in teaching math. The PMI of teachers may continue to strengthen through professional learning, gaining knowledge and efficacy, as well as lowering their anxiety. However, their beliefs toward teaching and learning math remain traditional. Despite strong math knowledge and confidence in one's teaching ability, a transmissive approach to math is firmly embraced. Choosing to implement this approach can stem from different pressures such as the demands of covering the curriculum (Darling-Hammond & Richardson, 2009), preparing for high-stakes tests (Boaler & Staples, 2008; Darling-Hammond & Richardson, 2009), or fear that the complexity of math is diluted through reform practices (Schoenfeld, 2004).

Furthermore, it is important to note that transmissive approaches to math instruction are often part of a school's culture, thereby perpetuating long-standing teaching practice (Hart, 2004). The instruction of teachers situated in this zone is dominated by algorithm dependency with a focus on procedures. This, in turn, does not promote a deep understanding of math ideas on a conceptual level. In attempts to engage teachers in the more dynamic approaches to teaching and learning, it is important to provide them with meaningful classroom examples and evidence-based research on how these pedagogies support their students' math achievement.

Discussion

The foundational concepts of the *Zones of Math Knowledge Creation Framework* takes into consideration the personal capacity and identity of teachers at any point in their career. Location within the framework is fluid, and will change as capacities and identities intersect differently with grade levels, students taught, curriculum content, and external pressures to implement specific pedagogical approaches. All teachers can increase their PMI as they evolve in their profession. As teachers increase knowledge, increase efficacy toward teaching and learning math, and lower levels of math anxiety, their readiness for engaging in dynamic approaches is more easily attainable.

Nevertheless, PMI does not automatically increase with expanded teaching experience as it is influenced by many factors. For example, veteran teachers may have taught in the primary grades for their entire careers, feeling very confident with the math knowledge they possess for the early grades. Yet, these same teachers may be asked to teach math at the middle school level thereby decreasing their teaching efficacy and increasing their anxiety due to the content knowledge that is required in the curriculum. In this situation, their PMI could be located on the left side of the continuum. Since math identities are fluid (Azmitia, Sye, & Radmacher, 2008) and confidence in math is dependent on so many variables, teachers will find themselves in different locations in the *Zones of Math Knowledge Creation* framework throughout their careers.

The future of math instruction is critical as society's demand for STEM roles increase (Ramsey & Baethe, 2013; Watt et al., 2017). Unfortunately, there will be a major gap in the workforce as potential positions outnumber the amount of people who possess the required math skills (Friedman, 2005; Lowell & Regets, 2006). So, how can school systems foster communities of mathematicians that include all students and teachers? Foundational to this work is transforming how students and teachers view this subject, as well as their relationship with this subject. Math must be regarded as interesting, challenging, and interactive, in which productive struggle to find solutions is the underpinning of learning itself. In fact, the ultimate goal should involve everyone seeing themselves as mathematicians, embracing challenging math problems and persevering until the problems are successfully unravelled, demystified, and conquered with excitement.

Within the *Zones of Math Knowledge Creation*, the *Launch Zone* is where many elementary teachers in Ontario might find themselves. Due to their math schooling experiences, transmissive approaches to teaching and learning have historically been the norm (Bruce, 2005; NCTM, 2014). Further, provincial data suggest that a majority have not encountered math in a significant way during undergraduate university. Based on EQAO questionnaires of grade three and six in-service teachers, approximately 80% possess a university major or minor that is unrelated to math (EQAO, 2018c; EQAO, 2018d). To support teachers situated in this zone, widespread professional learning and knowledge mobilization of math within and between classrooms, schools, districts, and beyond are necessary.

Researchers suggest that adult professional learning should occur as close to the classroom as possible, as well as being connected to the immediate learning needs of teachers (Reid, 2014; Merriam, 2001). In this zone, the learning needs of teachers are apparent, improved MCK, as well as MKT. For without solid knowledge base of content knowledge, teachers are challenged to engage appropriately with students who struggle with concepts and misconceptions (Ball et al., 2008), as well as making instructional decisions informed by student gaps in understanding (Lui & Bonner, 2016). Without a deeper understanding of the content knowledge, development of MKT and dynamic instructional approaches is threatened (Thames & Ball, 2010), resulting in a continued focus on procedural knowledge without conceptual understanding. Consequently, any attempts to achieve large-scale reform of instructional practice will necessitate sustained and systemic assistance.

Based on a continued prevalence of transmissive approaches to instruction (Banilower, Boyd, Pasley, & Weiss, 2006), it is probable that most secondary math teachers in Ontario would find themselves in the *Potential Zone*. Although they possess higher PMI due to stronger content knowledge and efficacy for teaching math, secondary math teachers may be reticent toward incorporating a dynamic approach to teaching and learning. In fact, providing procedures and algorithms can be viewed as a way to make math more accessible for students. However, without an understanding of the inner workings of the algorithm, math is prone to become a subject without reason (Nardi & Steward, 2003). Further, teachers can view activities such as the use of manipulatives as 'fun math', as compared to 'real math' involving transmissive approaches of textbooks and worksheets (Moyer, 2001). For a majority of math teachers, a transmissive approach worked for them in becoming a mathematician. Therefore, changes to their approach to teaching and learning can be perceived as detrimental to their own students (Charalambous & Philippou, 2010).

Teachers with lower levels of PMI may feel reticent to investigate deeper levels of dynamic teaching and learning. If guidance and support are afforded to teachers, the *Exploratory Zone* can be positively experienced. In this zone, teachers may gain confidence to challenge their own notions of instruction, as well as increase their PMI. Most importantly, when teachers are strategically assisted by others through learning networks, knowledge gained can be shared broadly across teams, schools, and systems (Katz, Earl, & Ben Jaafar, 2009). Conversely, teachers' learning that is unsupported most often results in experiences of isolation with an absence of occasions to engage with professionals or share new knowledge (Kim, 1993).

Additionally, teachers may feel an external pressure to implement dynamic approaches to teaching and learning without a supportive network in reach. For instance, teachers might present open-ended math problems to their students based on suggestions from superiors in their organizations. At the same time, teachers' math anxiety may increase as students develop unique responses to the problems in which teachers are unfamiliar with and cannot determine whether the students' strategies are accurate, applicable, or even valid. Without other teachers to confer with and exchange ideas, the pull toward providing traditional approaches may be strong. Unfortunately, teachers in this zone may feel ill prepared to experiment with more dynamic methods, perhaps sensing isolation in their practice due to a lack of formal or informal learning networks.

The *Optimal Zone* of math knowledge creation offers the most advantage for the teaching profession to be situated. Educational systems can support teachers in moving toward this zone by providing personalized and mobilized professional learning opportunities. This learning can contribute to concept attainment, collective sharing, knowledge creation, collaborative inquiry, and guided experimentation based on the math needs of their students. These conditions allow teachers to further develop both their MCK and MKT, bringing new learnings to the classroom for further exploration and reflection. Within this zone, teachers can feel confident to explore new concepts and pedagogical methods as they build their math competencies.

Historically, educational jurisdictions introduce instructional practices or new initiatives by providing significant funding for resources such as time for collaboration (Phillips, 2003). Regrettably, these vital start-up resources are almost always withdrawn as initiatives are expanded more broadly in organizations (Bulkley, Christman, Goertz, & Lawrence, 2010). It is not lost on the author of this paper that large-scale professional learning opportunities would be a costly endeavour, however, without a dedicated commitment toward sustained jurisdictional learning communities, the cycle of transmissive approaches to math instruction will remain unbroken.

Conclusion

Through the literature review, various supporting factors for this paper were established. The *Zones of Math Knowledge Creation* provides researchers with a framework to base future empirical studies upon. Overall, this paper advocates for extensive, sustained, and systemic professional learning to support transformative math environments. However, professional learning in and of itself will not support teachers if the concept of dynamic teaching and learning is not widely understood. This requires teachers of math, as well as curriculum designers, policy makers, and educational leaders to understand that dynamic approaches to math includes an attention to conceptual understanding, constructivist learning, along with procedural fluency. For instance, rules, procedures, and algorithms are not avoided in dynamic math classrooms. Instead, deep conceptual understanding and advanced flexibility with math ideas offer students opportunities to confidently apply algorithms when appropriate.

The author of this paper strongly believes that teachers desperately want their students to achieve in math, but may not know how to move forward based on their own history with math. Further, teachers may feel numerous pressures that impede their focus on math such as changing district or provincial initiatives, shifting demographics in the classroom, or lack of resources to engage students in deep learning of math. Therefore, organizations responsible for policy, curriculum, and professional learning must be clear with the ultimate approach to instruction and they must also open pathways for all teachers to move toward the preferred *Optimal Zone*.

Without the provision of growth pathways based on the strengths and needs of teachers, anxiety and guilt may be the unfortunate outcome thereby negatively influencing PMI. Just as teachers are charged with the responsibility to understand their students and plan accordingly, it is vital that educational systems consider the starting points and learning pathways of their teachers. For math instruction to transform, personal math identities and approaches to teaching and learning must move in a north-easterly direction in this framework. To occur systemically, it is necessary to immerse teachers in sustained environments of collaborative professional learning. Then and only then, effective instructional practices will be mobilized, networks of support and learning will be nurtured, and generations of mathematicians will be cultivated.

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