Mathematical Remedial Lesson Plan in Teaching Learning Process at University Level: Dihedral Group

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Introduction

In every university, learning of modern algebra for undergraduate students of mathematics is to the point and understanding the coursework is application based. Many researchers state that lecture-based mathematics instruction is unapproachable and confusing to students about the nature of mathematics, especially in modern algebra proof-based course for undergraduate and postgraduate students. It is believed by some lecturers in our mathematics department that the low performances of students opting this course is connected with the fact that progressing from the concrete situations of elementary algebra in high school to the more abstract algebra of undergraduate studies is usually a complicated transition for many students. Most students were able to demonstrate a modicum of aptitude in elementary algebra at high school, the majority fail to acquire a solid conceptual understanding of the abstract algebra offered at university level. The usual focus of mathematical activities in high school is on manipulations of symbols to find particular numerical answers. In contrast, abstract algebra requires students to think in terms of the structure of problems rather than in terms of operations. It involves procedures, interpreting and understanding relationships and expressing them in general simplified form. The following question now emerges: how could one improve students understanding of abstract algebra offered at university level? The author believes that any improvement in understanding could involve the incorporation of new lecture plan, such as general purpose and readily available basic logic of content in teaching. In this paper, our purpose is to sort out the problem of those students, who are pursuing their undergraduate, postgraduate and students who wish to do specialized education after a long gap or are coming from another stream. The purpose of this paper is to create a lesson plan on the specific topic dihedral group for those students who have not understood the basic logic of modern algebra. Here the author also discusses about situation that arises in the class room during delivering the lecture at post-graduation level, that the teacher instructs from the front of the room while the students sat in desk arranged to face the board, here the teacher is not aware about students previous knowledge regarding the subject as put up earlier that the students are based from different field or are perusing the degree after a long gap, resulting in no student participation in the classroom. As a result, student lags behind and creates high failure rate in the course.

Literature Review

The following section describes various aspects of the study in mathematical learning with a brief literature survey. (Goodchild, S., 2020) put forward a book on philosophy of mathematics education today. (Chen & Wu, 2020) studying how to effectively implement remedial instruction in mathematics education. Results of this paper showed that when ICT-integrated mathematics remedial instruction was not implemented, students' scores in the post test were not significantly higher; however, after implementing ICT-integrated mathematics remedial instruction, the grades in the post test were significantly higher. The results may facilitate the application of technology to implement mathematics remedial instruction. A survey on word problems in mathematics education had proposed by (Verschaffel et al., 2020). Word problems are among the most difficult kinds of problems that mathematics learner's encounter. Perhaps as a result, they have been the object of a tremendous amount research over the past 50 years. This opening article gives an overview of the research literature on word problems solving, by pointing to a number of major topics, questions, and debates that have dominated the field. Author (Verschaffel et al., 2020)., also review research on the impact of three important elements of the teaching-learning environment on the development of learners' word problem solving competence: textbooks, software, and teachers. The article (Erbilgin, 2019). investigates how collegial lesson preparation and reflection that focuses on prospective teachers' thoughts supported the participation of mathematics teacher educators' perspectives on teaching and learning. In this paper the data sources incorporated weekly planning and reflection meetings, activity worksheets, reflective journals, and audio recordings of lessons. Data analyses revealed that engaging in this action, research provides the contribution of teacher educators with opportunities, to discuss important issues for mathematics teacher educators and improved their skills in lesson planning and implementing. The interrelationship between Mathematics and development of humans to advance the cause of humans is a fundamental significance of Mathematics to humans has been described by (Brijlall, 2020). The author draws on many empirical studies which address quality in teaching and learning. The focus is to summarize many research studies which deal with the teaching and learning of Mathematics in Higher Education. Author (Cohen, 2018), point up that Education is a complex process involving teachers and learners within surrounding institutional and social environments over which neither group has extensive control. However, complexity is not an excuse for despair or inactivity. In particular, there is an attractive intellectual challenge involved in finding accessible and engaging ways to help teachers understand the mathematics itself. With the reference of article (Novak et al., 2017), author describes students' perspectives of a one-off flipped lecture in a large undergraduate mathematics service course. The focus was on calculating matrix determinants and was designed purposely to introduce debate and argumentation into a mathematics lecture. The

intention of the author (Novak et al., 2017) was to promote a deeper learning and understanding through engagement with the added hope of instilling some passion for the subject. Within this paper, authors (Novak et al., 2017) share the data and reveal the interesting results that emerged from their analysis. This article (Wasserman et al., 2016), explores the potential for aspects of abstract algebra to be influential for the teaching of school algebra. Author (Wasserman et al. 2016) using national standards for analysis, four primary areas common in school mathematics and their development across elementary, middle and secondary mathematics, where teaching may be transformed by teachers' knowledge of abstract algebra are developed. Author (Pimm et al. 2019) has presented an impressive book titled "Speaking Mathematically". The author with reference to piece of writing (Johnson et al., 2018) says, in the United States, there is significant interest from policy boards and funding agencies to change students' experiences in undergraduate mathematics classes. Author (Johnson et al., 2018) conducted a national survey of abstract algebra instructors at Master's and Doctorate-granting institutions in the United States to investigate teaching practices, to identify beliefs and contextual factors that support the constrain of non-lecture teaching practices, and to identify commonalities and differences between those who do and do not lecture. This work provides insight into how abstract algebra is taught in the United States, for how to approach and better support those who are interested in implementing non-lecture teaching approaches. In this article (Park et. Al., 2018) author says, developmental education became optional for many college students in Florida, regardless of prior academic preparation. This study investigated first-semester math course enrolment patterns for underprepared first-time-in-college students, who would have previously been required to take developmental math and the passing rates for the students electing to take Intermediate Algebra. Development of mathematical connection skills in a dynamic learning environment have described by (Zengin, 2019). Author's purpose of this study was to examine the effect of GeoGebra software on pre-service teachers' mathematical connection skills. After GeoGebra implementations, the quantitative data were analysed using a dependent t-test and the qualitative data obtained with the open-ended questionnaire were analysed using descriptive analysis. Based on the results, it was determined that GeoGebra software could be used as an important tool for the development of mathematical connection skills. Author (Orcos et al., 2019) describes the development of an application that allows compulsory secondary education teachers the assessment of the students' maths competence. The results of this paper (Orcos et al., 2019) gathered throughout speaking tasks, tests and work done by students in 2nd, 3rd and 4th course of compulsory Secondary Education of a state school in Spain. Research in article (Breen et al. 2019) has shown that the types of tasks assigned to students affect their learning. Various authors have described desirable features of mathematical tasks or of the activity they initiate. Others have suggested task taxonomies that might be used in classifying mathematical tasks. The masterpiece (Sinan & Zekeriya,

2018) related to the study of developing a lesson plan for the "Graphics and Animation in Education" course lectured in the department of Computer Education and Instructional Technology. The basic strategy used by author for this course is stated as "Expository" and during the course demonstration question-and-answer methods were used. Resulting author (Sinan & Zekeriya, 2018) developed a unit lesson plan for graphics and animation in education course. Article (Nuchanart et al., 2014) propose an experimental result with target group of students using 11 lesson plans for mainly focusing on development of lesson plans, development of the students thinking skills and study the student's opinions and also have an observation test form and achievements test with emphasis on thinking skills. The criteria with reference to article (Marcia & Gayani 2010), author explains that effective teaching in higher education is understood to comprise particular skills and practices applied within particular contexts. The paper maintains that our collective understanding of competent, professional and effective teaching must repeatedly evolve in order that it accurately reflects and responds to the contexts in which learning and teaching is undertaken. The author referencing article (Saa et al., 2019) expresses his view that, predicting the students' performance has become a challenging task due to the increasing amount of data in educational systems and also showed that the most common factors are grouped under four main categories, namely students' previous grades and class performance, students' e-Learning activity, students' demographics, and students' social information. In this paper, authors have presented a comprehensive survey based on the student's frail learning on selected topic "Dihedral Group". The rest of the paper is organized as follows. Section 2 gives an overview of the survey on problem analysis throughout the lecture. The design of remedial lesson plan is presented in section 3. Section 4, is associated to evaluation and discussion. Conclusion with acknowledge and references are given in section 5.

Problem Analysis During Lecture Process

Any lecturer organises and presents a lecture of abstract algebra on the topic: dihedral group. "A dihedral group is a group of symmetries of regular polygon which involves rotations and reflections. The dihedral group is an example of a finite group and plays an important role in group theory, geometry and chemistry. Dihedral group basically represented by, is a group of order 2n. A regular polygon with n sides has different symmetries i.e. rotational symmetries and reflection symmetries." After introducing the topic during the lecture few questions were asked to the students to know their understanding. The three different questions were break-down on the aspects as: previous knowledge, about group and order of the group.

After delivering the lecture on the topic dihedral group, the lecturer asked some questions to the students for evaluating their previous knowledge and understanding level.

Lecturer: Why (a set of all rotational and reflection of a polygon) is said a dihedral group?

Student: Because the set satisfies all properties of a group.

Lecturer: What is a regular polygon?

Student: A regular polygon is a closed figure with all sides of same length and all angles are of equal measurement.

Lecturer: (To another student) what properties are necessary for a set to be a group?

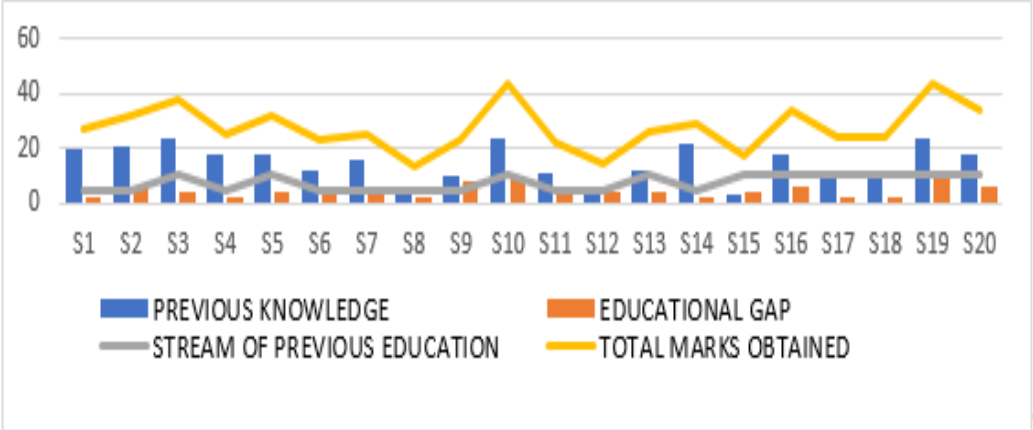
(Out of 20 students only 14 students gave right answer other didn't know about the group).

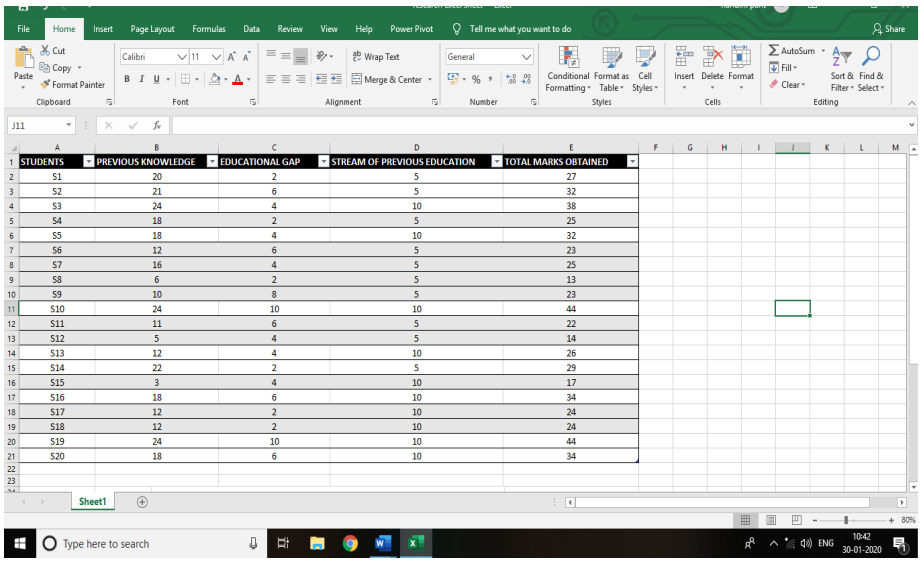
Lecturer: What is the order of a group?

(Only 11 students gave correct answer).

Now here a question strikes to the teacher that why all students are unable to answer questions based on basic Morden algebra and group theory. Before delivering the lecture, lecturer plans to take a survey to know the actual problem of those students who were unable to understand the basic concept based on modern algebra. Here the lecturer has organised a survey with 20 students to get clear idea on aspects namely pervious knowledge regarding the topic, education qualification of the student and student's education stream, on bases of their marks and percentage which is stated in table 1. Based on these aspects the lecturer will record student's marks as per there scoring out of 24 marks in the first aspect i.e. previous knowledge. With the second aspect the evaluation of marks are on the bases of there education gap, as the students with gap of 00-01 years is given 10 marks, students with gap of 00-02 years is given 8 marks, students with gap of 00-04 years is given 6 marks, year gap with 00-06 is given 4 marks, year gap with 00 -08 is given 2 marks and lastly year gap of 00-10 years is given 0 marks. Lecturer takes the 3rd condition for evaluation i.e. students coming from different stream of education, are categories in two ways i.e. students with same stream are given 10 marks and students with different stream are given 05 marks. Later on, based on total marks of each student. Lecturer has done the total evaluation and has planned remedial lesson plan for student's scoring 25 marks and below. The survey helps to design a good remedial plan for student's betterment and easy learning of the topic. Here the table 1 and the survey graph figure 1 gives us the clear understanding:

Table 1. Students Survey Table.





STUDENTS	PREVIOUS KNOWLEDGE	EDUCATIONAL GAP	STREAM OF PREVIOUS EDUCATION	TOTAL MARKS OBTAINED
S1	20	2	5	27
S2	21	6	5	32
S3	24	4	10	38
S4	18	2	5	25
S5	18	4	10	32
S6	12	6	5	23
S7	16	4	5	25
S8	6	2	5	13
S9	10	8	5	23
S10	24	10	10	44
S11	11	6	5	22
S12	5	4	5	14
S13	12	4	10	26
S14	22	2	5	29
S15	3	4	10	17
S16	18	6	10	34
S17	12	2	10	24
S18	12	2	10	24
S19	24	10	10	44
S20	18	6	10	34

Figure 1. Survey Graph

Design of the Lesson Plan on Dihedral Group with Graphical and Example Based Approach

Dear students, first we all should know about various basic terms which are important for learning of dihedral group. But before we start are topic, we will see those terms that will help us comprehend our topic more evidently.

Symmetry

In mathematics, symmetry is a property of an object that is invariant under some transformations, including translation. reflection and rotation or scaling.

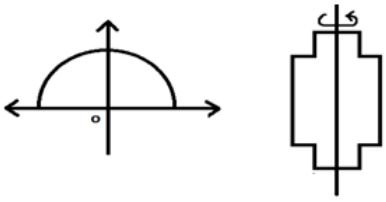


Figure 2. Symmetry About the Horizontal Axis.

A geometric object is symmetric if it divides two or more identical pieces that are arranged in an organized approach. Both semi-circle and blade in fig. 2 are symmetric about horizontal axis.

Reflection symmetry

Reflection symmetry is a type of symmetry which is with respect to reflections. Reflection symmetry is also known as line symmetry or mirror symmetry. If there exists at least one line that divides a figure into two halves such that one-half is the mirror image of the other half.

- The line where a mirror can be kept so that one-half appears as the reflection of the other is called the line of symmetry.
- A figure can have one or more lines of reflection symmetry.
- The line of symmetry can be in any direction.

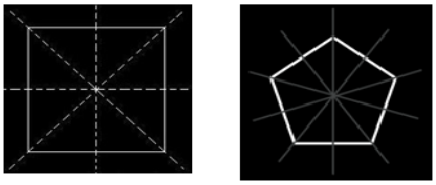


Figure 3. Line of ″ymmetry in any µirection".

Rotational Symmetry

If a figure is rotated around a center point and it still appears exactly as it did before the rotation, it is said to have rotational symmetry. A number of shapes like squares, circles, regular hexagon, etc. have rotational symmetry.

There are many shapes you will see in geometry which are symmetrical rotationally, such as:

- Equilateral triangles, Squares, Rectangles, Circles, Regular Polygons

Order of Rotational Symmetry

The number of positions in which a figure can be rotated and still appears exactly as it did before the rotation, is called the order of symmetry. For example in fig 4, a star can be rotated 5 times along its tip and look at the same every time. Hence, its order of symmetry is 5. Similarly, the recycle logo has an order of symmetry of 3 and the triangle has an order of symmetry of 3.

Figure 4. Order of Rotational Symmetry

Regular polygon

A regular polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length) or if all the sides and interior angles of the polygon are equal, then it is known as a regular polygon.

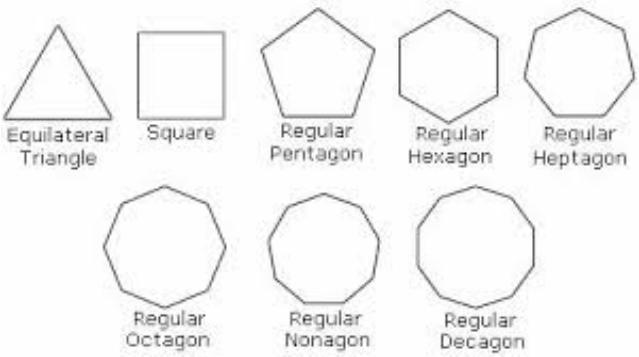


Figure 5. Regular Polygon

Group

A set equipped with a binary operation G= {S, o} where S is an non-empty set and o is operation, is called group if it satisfies four properties as,

a. Closure property: a o b = c ∀ a, b, c ∈ S.
b. Associative property: a o (b o c) = (a o b) o c.
c. Existence of identity: a o I = a = i o a ∀ a, i ∈ S, where I is an identity element.
   Identity element is an element of a group, which leaves any element at the set unchanged when combined with it.
d. Existence of inverse: a o a^{-1} = i = a^{-1} o a.

Symmetric Group

The symmetric group is the group of permutations on *n* objects. The group operation on is composition of functions. In simple words we find a group of symmetric shape by rotating and flipping the pattern known as symmetric group and is denoted by (Gallian, 2010).

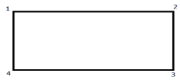


Figure 6. Symmetry of Rectangle

Here as we can observe that this rectangular has equal angles but does'nt have equal sides. So only by flipping the rectangluar horizontally and vertically we get to see its symmetry. Seeing fig.(7) flipping the rectanglur pattern, we get the same symmtery.

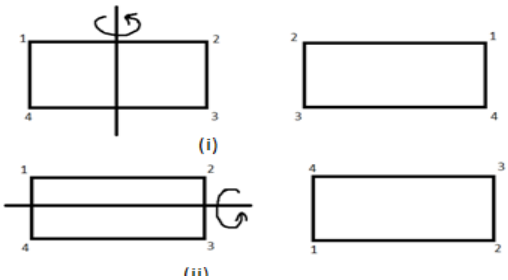


Figure 7. Vertical (i) & Horizontal (ii) Flipping

According above flipping of rectangle we get a set of all possible symmetry:

$$S_1 = \left\{ \begin{pmatrix} 1 & 2 & 4 & 3 \\ 2 & 1 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 & 3 \\ 4 & 3 & 1 & 2 \end{pmatrix} \right\}$$

Let us take another example of an arrow pattern in which by flipping (reflection), the pattern here only one symmetry is obtained.

Figure 8. Reflection of Arrow

$$S_2 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 1 & 5 & 4 & 7 & 6 \end{pmatrix} \right\}$$

is the set of the pattern and only two elements are possible for this pattern. As every angle is different in this, so as a result there is no angular rotation.

Theoretical Framework of Dihedral Group

The group of symmetries of an n-sided regular polygon for $n > 3$ with rotations and reflections is termed Dihedral group, which is denoted as D_n. The word dihedral means two phases, and if we have a polyhedral it means many phases. The symmetry of the dihedral group depends on two types of symmetries (i) Rotation about an angle, such as figure looks same, (ii) Flipping (i.e. reflections) about the sides. The rotation of an angle depends on number of sides a regular polygon i.e. the size of angle of rotation = $\frac{360^0}{n}$ radian, where n is the number of sides a regular polygon (Gallian, 2010).

$$\text{Where } D_n = \left\{ \begin{matrix} x, x^2 = e \\ y^1, y^2, \ldots\ldots, y^{n-1} \\ xy^1, xy^2, \ldots\ldots, xy^{n-1} \end{matrix} \right\}$$

The order of the Dihedral group is $2n$. It is also known as permutation group. The set of all possible symmetry of equilateral triangle is known as dihedral group D_3. The dihedral group D_3 has order 6=(2(3)), which is found by six symmetries of an equilateral triangle.

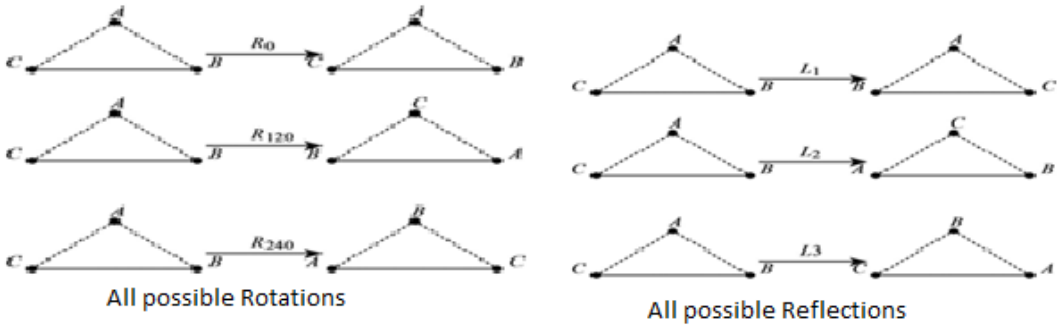


Figure 9. All Possible Ways of Rotations and Reflections for Equilateral Triangle.

The multiplication-table of D_3 is:

$(D_3,0)$	R_0	R_{120}	R_{240}	L_1	L_2	L_3
R_0	R_0	R_{120}	R_{240}	L_1	L_2	L_3
R_{120}	R_{120}	R_{240}	R_0	L_3	L_1	L_2
R_{240}	R_{240}	R_0	R_{120}	L_2	L_3	L_1
L_1	L_1	L_2	L_3	R_0	R_{120}	R_{240}
L_2	L_2	L_3	L_1	R_{240}	R_0	R_{120}
L_3	L_3	L_1	L_2	R_{120}	R_{240}	R_0

Order of Dihedral Group

The dihedral group is denoted as D_n or D_{2n} and is generated by two elements. We can basically represent these elements by x and y, in which x is one element of order 2 and y is another element of order n. We can understand this condition by eq. (1). On the bases of element order of dihedral group, the relation defined is $D^n=\{x,y : x^2=e, yx=xy^{-1}\}$

Symmetric Group Generated by a Square

The eight transformations of a square shown below form the dihedral group D_4 with 8 elements. Transformations 2, 3, and 4 are counter clockwise rotations by 90°, 180°, and 270° respectively. Transformations 5 and 6 are vertical and horizontal reflections, while transformations 7 and 8 are reflections across the two diagonals of the square. Considering the examples of a regular polygon i.e. square, which symmetries set shows a group i.e. dihedral group of order 8. As we know that a square has four sides and angle between each adjoint sides are 90 degree, therefore we can find the angular rotation and reflections of this pattern.

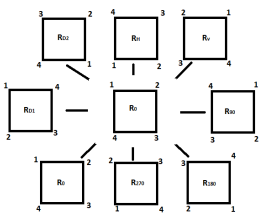


Figure 10. All Possible Symmetry of a Square

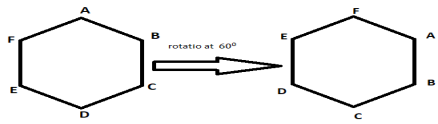
Initially we will first rotate this square at 90° degree i.e. $R_0^0 \rightarrow R90^o$ and the element is R_{90}^o. Second angular rotation in 180° is $R_0^o \rightarrow R_{180}^o$, third is $R_0^o \rightarrow R_{270}^o$. On rotating $R_0^o \rightarrow R_{360}^o$, we get our initial patter i.e. R_0^0. Hence we will now flip (reflection) the pattern with respect to principle diagnoal i.e. D_1 , Reflection with respect to second principle diagnoal is D_2 , Refelection of the pattern horzontally i.e. R_H and Reflection on vertical axes is R_Y. Symmetry of rectangluar polygon is

$$D_4 = \left\{ \begin{matrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 & 3 \\ 4 & 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 & 3 \\ 3 & 4 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, \\ \begin{pmatrix} 1 & 2 & 4 & 3 \\ 3 & 2 & 4 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 3 & 2 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix} \end{matrix} \right\}$$

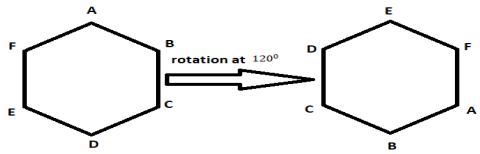
S is the set that denotes all the possible elements of rectangular polygon, and values of this polygon is written as = { I , (1 4 3 2) ,(1 3) (2 4) , (1 2 3 4) , (1 3) (2) (4), (1)(3)(2 4), (1 4) (2 3) ,(1 2) (3 4) }

(D_4, o)	R_0^0	R_{90}^0	R_{180}^0	R_{270}^0	R_{D1}^0	R_{D2}^0	R_H^0	R_V^0
R_0^0	R_0^o	R_{90}^o	R_{180}^o	R_{270}^o	R_{D1}^o	R_{D2}^o	R_H^o	R_V^o
R_{90}^0	R_{90}^o	R_{180}^o	R_{270}^o	R_0^o	R_V^o	R_H^o	R_{D1}^o	R_{D2}^o
R_{180}^0	R_{180}^o	R_{270}^o	R_0^o	R_{90}^o	R_{D2}^o	R_{D1}^o	R_V^o	R_H^o
R_{270}^0	R_{270}^o	R_0^o	R_{90}^o	R_{180}^o	R_H^o	R_V^o	R_{D2}^o	R_{D1}^o
R_{D1}^0	R_{D1}^o	R_H^o	R_{D2}^o	R_V^o	R_0^o	R_{180}^o	R_{90}^o	R_{270}^o
R_{D2}^0	R_{D2}^o	R_V^o	R_{D1}^o	R_H^o	R_{180}^o	R_0^o	R_{270}^o	R_{90}^o
R_H^0	R_H^o	R_{D2}^o	R_V^o	R_{D1}^o	R_{270}^o	R_{90}^o	R_0^o	R_{180}^o
R_V^0	R_V^o	R_{D1}^o	R_H^o	R_{D2}^o	R_{90}^o	R_{270}^o	R_{180}^o	R_0^o

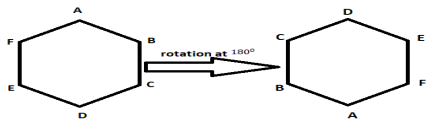
Example based on order of dihedral group of a regular polygon :



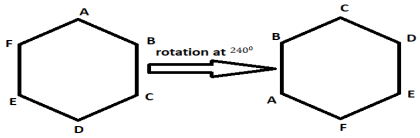
So rotation of R_0^0 polygon at an angle of 60^0 i.e. $R_0^0 \rightarrow R_{60}^0$



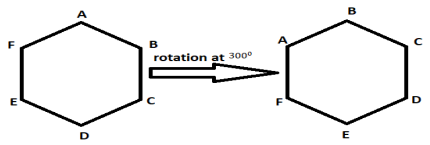
Rotation of R_0^0 polygon at an angle of 120^0 i.e. $R_0^0 \rightarrow R_{120}^0$



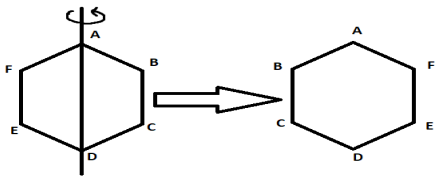
Rotation of R_0^0 polygon at 180^0 i.e. $R_0^0 \rightarrow R_{180}^0$



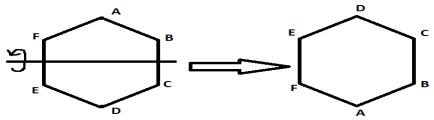
Rotation of R_0^0 polygon at an angle of 240^0 i.e $R_0^0 \rightarrow R_{240}^0$



Rotation of R_0^0 polygon at an angle of 300^0 i.e. $R_0^0 \rightarrow R_{300}^0$



Reflection of R_0^0 polygon at vertical axis i.e. $R_0^0 \rightarrow R_V$.



Reflection of R_0^0 polygon at horizontal axis i.e. $R_0^0 \rightarrow R_H$.Therefore, the reflection and rotation set/order formed for a rectangular polygon is:

$$S = \{R_0^0, R_{60}^0, R_{120}^0, R_{180}^0, R_{240}^0, R_{300}^0, R_V, R_V.R_{60}^0, R_V.R_{120}^0, R_V.R_{180}^0, R_V.R_{240}^0, R_V.R_{300}^0\}$$

Evaluation

Commonly evaluation process of any course is conducted at the end of semester or the topic. As in the case of remedial lesson plan, our assessment process is conducted at the completion of our topic with a different logical aspect. Based on the learning outcome of the topic, lecturer categorizes few questions for students in two parts as one question related to topic and other is application based which is necessary to evaluate at higher education level. We observe that after delivering such type of remedial lesson plan, the knowledge of student had improved then what it was earlier.

Discussion

Many researchers work on children mathematical learning disabilities. Basically, most of the researcher works on mathematical learning disabilities for the students of classes as primary, basic and secondary level. Author (Mazzocco et al., 2011), hypothesizes that mathematical learning disabilities partly results from a deficiency in the approximate number system that supports nonverbal numerical representations across species and throughout development. Author (Jitendra et al. 2020) investigated, taking sample of 338 students of seventh-grade with mathematical learning disabilities, based on scoring below 25th percentile on a proportional problem-solving pre-test. (Chu et al. 2019) Studied that in a preschool through first grade longitudinal study, author identified groups of children with persistently low mathematics achievement and children with low

achievement in preschool but average achievement in first grade. Author (Reeve et al. 2019), presented a review on math learning difficulties in Australia. The article (Wu et al., 2019), proposed a study on mathematical learning disabilities in primary school children. In (Wu et al., 2019), author aims to conduct training to improve the brain's cognitive ability for mathematics learning by focusing on two important mathematical cognitive abilities. Author (Lewis, 2017), presented a case study of a student with a mathematical learning disability for whom standard instruction on fractions had been ineffective. The entire above cited masterwork, shows that most of the study on mathematical learning disability was focused on students of primary and secondary level. Our work reveals the problem of mathematical learning disability in higher education level. We observed a group of students at graduation level, who are not affected with mathematical learning disability but they are unable to comprehend the topic under abstract algebra, which is included in their syllabus. Our study shows that if we are unaware of student's present knowledge related to the topic then one might not be able to make them clear the concept interrelated to the subject. After taking a survey on different parameters, our observation sticks to the point for those students who require a remedial course work and also a conceptual based remedial lesson plan not only at secondary level but also at higher level classes. Such type of courses which are not focusing on the shortcomings of mathematical learning, in other terms it is not less than a motiveless -unplanned lecture. This study also represent that every frail mathematical learning student is not categorised in the case of mathematical learning disability. Our study also reveals that such type of survey is a necessity not only for higher classes but also for middle and secondary classes, before starting the syllabus.

Conclusion and Future Work

We conclude the paper on a speculative note. Before starting a lecture, new topic and syllabus in university level first the lecturer should take a glance on the points discussed in this paper. Surprisingly students are unaware about basic of abstract algebra even after being from mathematics surroundings, the state of students was found after completion of this survey, if this survey is not taken in consideration before delivering any lecture then it might not be a productive lecture for students. Therefore, it is necessary to create a remedial lesson plan for those students in order to assist them in right direction. Moreover, we presented a lecture plan with the help of theory, graphical and example-based approach on the topic dihedral group keeping in mind all the aspects discussed above. Finally, we have used an evaluation process to show the feasibility and validity of remedial lesson plan. In further, it is very much necessary to solve such problem of students at higher level, not only with a good remedial lesson plan but also with a durable remedial syllabus (course work). In addition, the aspects discussed can be considered to develop a questioner not only for one subject but for every subject so as to have a fruitful outcome at the end of the lecture.

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